# EXAM 3

Math 212, 2016-2017 Spring, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name ______________________________

"I have adhered to the Duke Community
Standard in completing this
examination."

Signature: ____________________

1. __________

2. __________

3. __________

4. __________

5. __________

Total Score __________ (/100 points)

1. *(22 pts)* The curve $C$ is parametrized by $\vec{x}(t) = (\cos t, \sin t, t^2/\pi^2)$, with $t \in [0, 2\pi]$. The vector field $\vec{F}$ is given by $\vec{F}(x, y, z) = (\sin(yz), xz\cos(yz), xy\cos(yz))$.

   Compute the line integral $\int_C \vec{F} \cdot d\vec{x}$.

2. *(22 pts)* The path $C$ consists of several line segments connecting vertices (in the order of the orientation on the curve, and intersecting only at the vertices) $(10, 0, 0)$, $(1, 3, 0)$, $(5, 1, 1)$, $(6, 0, 2)$, $(12, -2, 2)$, $(16, -2, 0)$, and then back to $(10, 0, 0)$. The vector field $\vec{F}$ is given by $\vec{F}(x, y, z) = (3x - y - z, 2x + y, x - 3z)$.

   Compute the line integral $\int_C \vec{F} \cdot d\vec{x}$. *(Hint: Note that all of the vertices of $C$ are on the plane with equation $x + 3y + 2z = 10$.)*

3. *(22 pts)* The surface $S$ is the ellipsoid with equation $x^2 + 4y^2 + z^2 = 36$, oriented such that $\vec{n} = (1, 0, 0)$ at the point $(-6, 0, 0)$.

The vector field $\vec{F}$ is given by $\vec{F}(x, y, z) = (x^2, \cos(x^3) + 3y, z^2 - 2xz)$.

Compute the surface integral $\iint_S \vec{F} \cdot \vec{n}\, dS$.

4. *(12 pts)* The surface $S$ is the sphere of radius 3 centered at the origin, oriented toward the origin.

The vector field $\vec{F}$ is given by $\vec{F}(x, y, z) = \left( \frac{ye^{x-z}}{1+y^2}, \frac{-xe^{x-z}}{1+y^2}, 0 \right)$.

Compute the surface integral $\iint_S \vec{F} \cdot \vec{n}\, dS$.

5. *(22 pts)* The curve $C$ is the circle in the $xy$-plane of radius 2 centered at $(3, 0, 0)$. A torus (doughnut shaped surface) $T$ is created by rotating $C$ around the $y$-axis, and the surface $S$ is the part of $T$ with $z \geq 0$, oriented away from the "inside" of the doughnut.

   The vector field $\vec{F}$ is given by $\vec{F}(x, y, z) = (2 - xy^2, 5 - x^2y, 3 + z(x^2 + y^2))$.

   Compute the surface integral $\iint_S \vec{F} \cdot \vec{n}\, dS$.