EXAM 2

Math 212, 2016-2017 Spring, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name _____

"I have adhered to the Duke Community
Standard in completing this
examination."

1. _____

2. _____   Signature: _____

3. _____

4. _____

5. _____

Total Score _____ (/100 points)

1. *(20 pts)* In the vicinity of a certain mountain top, altitude H (measured in meters) is dependent on map coordinates (x, y) (measured in meters east and north (respectively) of a fixed point) as indicated by the equation below.
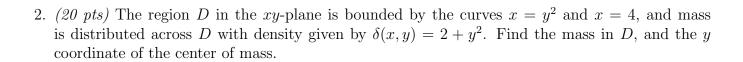
$$H(x, y) = 1000 - x^2 + xy - 4y^2$$

(a) Find a vector perpendicular to the contour line (level curve of the altitude function) at the point $(1, 2)$.
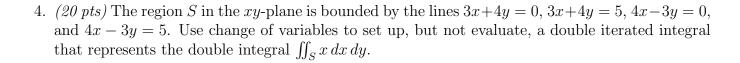
(b) At 2pm, Bob is at the point $(1, 2)$ and walking such that the velocity of his map coordinates is $(12, 5)$. What is the time rate of change of his altitude?

(c) How steep is the mountain at Bob's location at 2pm?

(d) How steep (dH/ds) is the Bob's path at 2pm? (This is positive if he is going uphill, negative if he is going downhill.)

2. *(20 pts)* The region D in the xy-plane is bounded by the curves $x = y^2$ and $x = 4$, and mass is distributed across D with density given by $\delta(x,y) = 2 + y^2$. Find the mass in D, and the y coordinate of the center of mass.

3. *(20 pts)* The region R in xyz-space is bounded by the surfaces with equations $x^2 + (y-1)^2 = 4$, $x^2 + y^2 + z = 16$, and $z = 0$. Mass is distributed through R with $\delta(x,y,z) = e^z$. Set up, but do not evaluate, a triple integral representing the moment of inertia of the mass in R around the y-axis.

Moment of inertia about the y-axis uses the squared distance $x^2 + z^2$:

$$I_y = \iiint_R (x^2 + z^2)\, e^z \, dV$$

Using cylindrical coordinates $x = r\cos\theta$, $y = r\sin\theta$, $z = z$:

- Top surface (paraboloid): $z = 16 - x^2 - y^2 = 16 - r^2$
- Bottom: $z = 0$
- Cylinder $x^2 + (y-1)^2 = 4 \Rightarrow r^2 - 2r\sin\theta - 3 = 0 \Rightarrow r = \sin\theta + \sqrt{\sin^2\theta + 3}$

$$I_y = \int_0^{2\pi} \int_0^{\sin\theta + \sqrt{\sin^2\theta + 3}} \int_0^{16 - r^2} \left(r^2\cos^2\theta + z^2\right) e^z \, r \, dz \, dr \, d\theta$$

4. *(20 pts)* The region S in the xy-plane is bounded by the lines $3x+4y=0$, $3x+4y=5$, $4x-3y=0$, and $4x-3y=5$. Use change of variables to set up, but not evaluate, a double iterated integral that represents the double integral $\iint_S x\,dx\,dy$.

5. *(20 pts)* The region T in xyz-space is the collection of points with $x^2+y^2+z^2 \le 9$, $z \le \sqrt{x^2+y^2}$, and $z \ge 0$. Set up, but do not evaluate, a triple iterated integral in spherical coordinates representing $\iiint_T z \, dV$.

$$\iiint_T z \, dV = \int_0^{2\pi} \int_{\pi/4}^{\pi/2} \int_0^3 (\rho\cos\varphi)\, \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$$