EXAM 2

Math 212, 2016-2017 Fall, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name Solutions

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ____________________

1. __________

2. __________

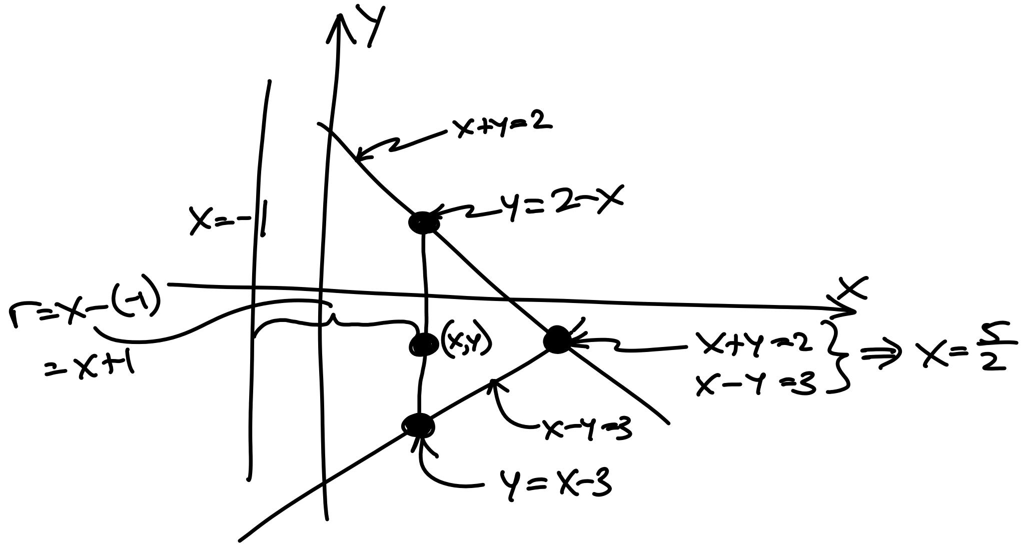
3. __________

4. __________

5. __________

Total Score __________ (/100 points)

1. *(15 pts)* A metal sheet occupies the region R of the xy-plane that is bounded by $x + y = 2$, $x - y = 3$, and $x = 0$. The density of the sheet is given by $\delta(x, y) = 5 + y$. Set up, but do not evaluate, an iterated integral representing the moment of inertia of this sheet around the line $x = -1$.



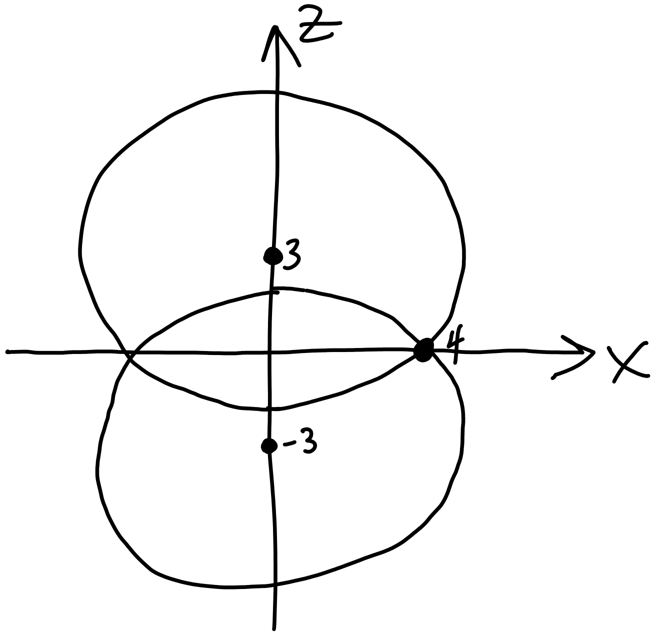
$$I = \iint_R r^2 \, dm$$

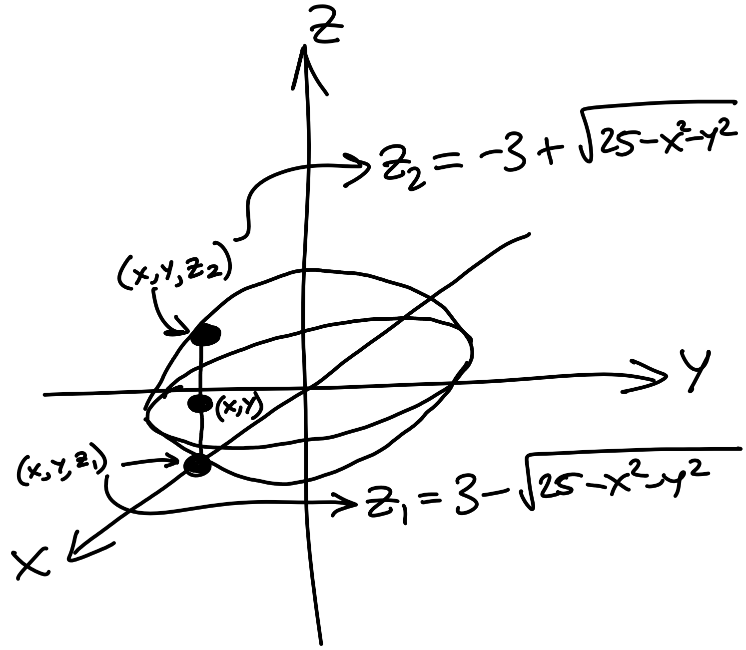
$$= \iint_R (x+1)^2 (5+y) \, dy \, dx$$

$$= \int_0^{5/2} \int_{x-3}^{2-x} (x+1)^2 (5+y) \, dy \, dx$$

2. *(20 pts)* The ball B_1 has radius 5 and center at $(0,0,3)$, and the ball B_2 has radius 5 and center at $(0,0,-3)$. The solid D is the intersection of these two balls.

Set up, but do not evaluate, a triple iterated integral in rectangular coordinates representing $\iiint_D f(x,y,z)\,dV$.



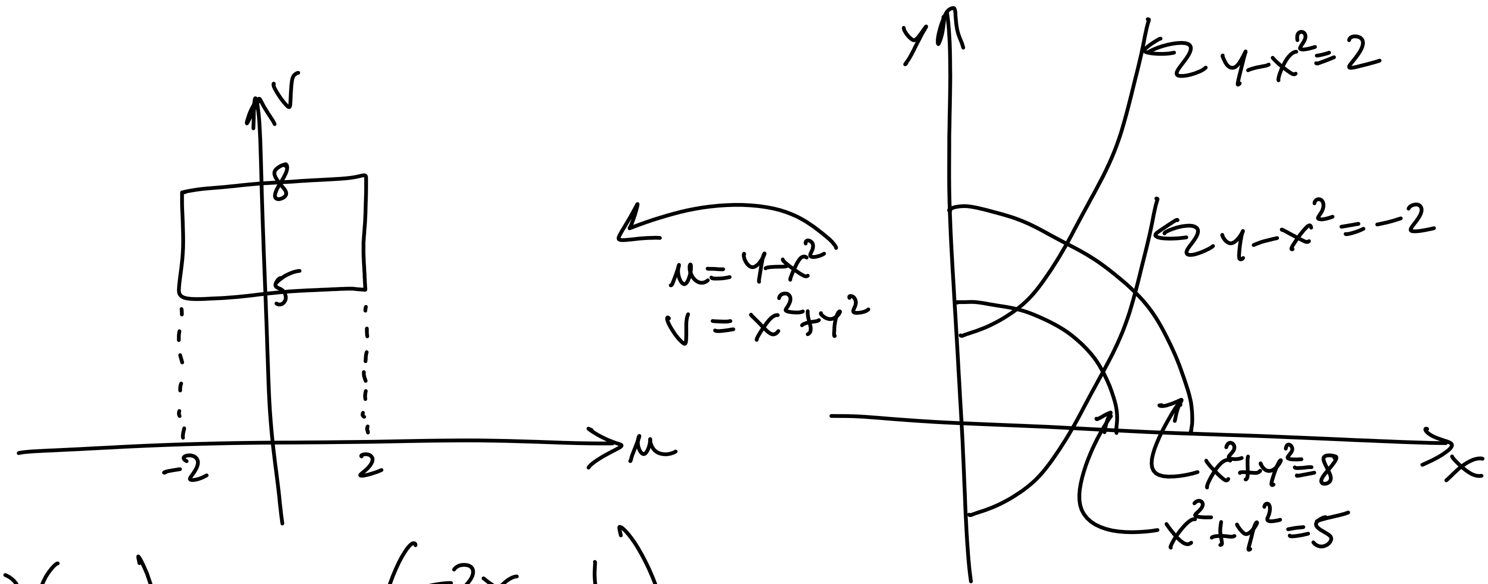


$$\iiint_D f\,dV = \underbrace{\int_{-4}^{4}\int_{-\sqrt{16-y^2}}^{\sqrt{16-y^2}}}_{\text{b/c proj. to xy-plane is disk of rad}=4}\int_{3-\sqrt{25-x^2-y^2}}^{-3+\sqrt{25-x^2-y^2}} f(x,y,z)\,dz\,dx\,dy$$

from top part of lower sphere

from bottom part of upper sphere

3. *(25 pts)* The region R in the first quadrant of the xy-plane is bounded by the curves $y = x^2 + 2$, $y = x^2 - 2$, $x^2 + y^2 = 5$, and $x^2 + y^2 = 8$. Compute $\iint_R (4xy + 2x)(y^2 + y)\, dx\, dy$.



$$\frac{\partial(u,v)}{\partial(x,y)} = \det\begin{pmatrix} -2x & 1 \\ 2x & 2y \end{pmatrix} = -4xy - 2x$$

$$\frac{\partial(x,y)}{\partial(u,v)} = \frac{1}{\partial(u,v)/\partial(x,y)} = \frac{1}{-4xy-2x}$$

This is < 0 because $x, y > 0$, so $\left|\frac{\partial(x,y)}{\partial(u,v)}\right| = \frac{1}{4xy+2x}$

Then

$$\iint_R (4xy+2x)(y^2+y)\,dx\,dy$$

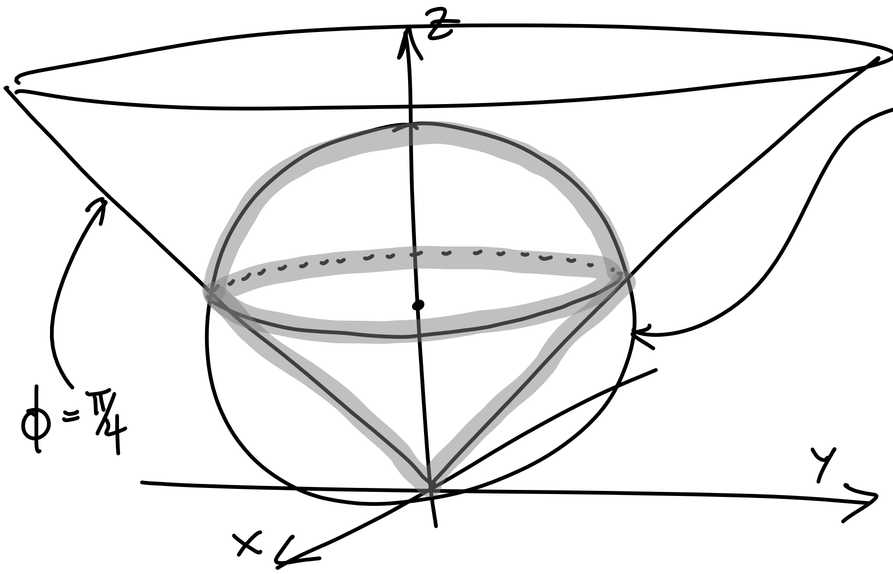
$$= \iint (4xy+2x)(y^2+y)\frac{1}{4xy+2x}\,du\,dv$$

$$= \int_5^8 \int_{-2}^{2} (u+v)\,du\,dv$$

$$= \int_5^8 \left(\tfrac{1}{2}u^2 + uv\right]_{u=-2}^{u=2} dv = \int_5^8 4v\,dv$$

$$= \left(2v^2\right]_5^8 = 78$$

4. *(20 pts)* The solid T is the region consisting of points (x, y, z) that are inside of the ball of radius 1 centered at $(0, 0, 1)$, and above the half-cone with equation $z = \sqrt{x^2 + y^2}$. Set up, but do not evaluate, a triple iterated integral in spherical coordinates representing $\iiint_R x\, dV$.



$$x^2 + y^2 + (z-1)^2 = 1^2$$

$$\rho^2 - 2z + 1 = 1$$

$$\rho^2 = 2\rho\cos\phi$$

$$\rho = 2\cos\phi$$

$$\iiint_R x\, dV = \int_0^{2\pi}\int_0^{\pi/4}\int_0^{2\cos\phi} (\rho\sin\phi\cos\theta)\left(\rho^2\sin\phi\, d\rho\, d\phi\, d\theta\right)$$

$$= \int_0^{2\pi}\int_0^{\pi/4}\int_0^{2\cos\phi} \rho^3\sin^2\phi\cos\theta\; d\rho\, d\phi\, d\theta$$

5. *(20 pts)* The surface S is the part of $x^2 - y^2 - z + 5 = 0$ that is above the rectangle $[0,1] \times [1,2]$ in the xy-plane.

Use a parametrization of S to set up, but do not evaluate, an iterated integral representing the surface area of S.

$z = x^2 - y^2 + 5 \Rightarrow S$ param. by $x = u$, $y = v$, $z = u^2 - v^2 + 5$, $u \in [0,1]$, $v \in [1,2]$

$$\vec{X}_u = \begin{pmatrix} 1 \\ 0 \\ 2u \end{pmatrix} \quad \vec{X}_v = \begin{pmatrix} 0 \\ 1 \\ -2v \end{pmatrix} \quad \vec{N} = (-2u, 2v, 1)$$

$$\|\vec{N}\| = \sqrt{1 + 4u^2 + 4v^2}$$

$$\text{Surf. area} = \iint_S 1 \, dS = \int_1^2 \int_0^1 (1) \sqrt{1 + 4u^2 + 4v^2} \, du \, dv$$