EXAM 3

Math 212, 2015-2016 Spring, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name _____

"I have adhered to the Duke Community
Standard in completing this
examination."

1. _____

2. _____

Signature: _____

3. _____

4. _____

5. _____

Total Score _____ (/100 points)

1. *(20 pts)* The curve C is parametrized by $\vec{r}(t) = (3\cos(\pi t), e^{t^2-t}, 2+t^3)$, with $t \in [0,1]$. A particle moving along C is acted on by a force field represented by $\vec{F}(x,y,z) = (yz, xz - z^2, xy - 2yz)$. Compute the amount of work that it takes for a particle to move along the curve C.

Solution:

Check if $\vec{F}$ is conservative. Seek potential f with $\nabla f = \vec{F}$:
$$f_x = yz \implies f = xyz + g(y,z)$$
$$f_y = xz + g_y = xz - z^2 \implies g_y = -z^2 \implies g = -yz^2 + h(z)$$
$$f_z = xy - 2yz + h'(z) = xy - 2yz \implies h'(z) = 0$$

So $f = xyz - yz^2$.

Endpoints:
- $t=0$: $x = 3\cos 0 = 3$, $y = e^0 = 1$, $z = 2$.
- $t=1$: $x = 3\cos\pi = -3$, $y = e^{0} = 1$, $z = 3$.

$$f(3,1,2) = 3\cdot1\cdot2 - 1\cdot4 = 2$$
$$f(-3,1,3) = (-3)(1)(3) - 1\cdot9 = -18$$

$$W = f(\text{end}) - f(\text{start}) = -18 - 2 = -20$$

2. *(20 pts)* The solid rectangle R_1 has corners at $(0,0,0)$, $(3,0,0)$, $(0,0,2)$, $(3,0,2)$ and is oriented in the positive y-direction, and the solid rectangle R_2 has corners at $(0,0,0)$, $(3,0,0)$, $(0,1,0)$, $(3,1,0)$ and is oriented in the positive z-direction. The surface S is the union of these two rectangles, with the same orientations, and the curve C is the boundary of S.

Compute the line integral of the vector field $\vec{F}(x,y,z) = (x+y-z, 2x-y+z, -x-y+z)$ along C.

3. *(20 pts)* The surface S is parametrized by $\vec{x}(\phi, \theta) = (2\sin\phi\cos\theta, 2\sin\phi\sin\theta, 2\cos\phi)$, where $\phi \in [0, \pi]$, $\theta \in [0, 2\pi]$. At the point on S with $\phi = \pi$ and $\theta = 0$, the oriented unit normal vector is $\vec{n} = (0, 0, 1)$. Compute the flux through S of the vector field $\vec{F}(x, y, z) = (2x - y^2, e^x - 2yz, xy + z)$. *(Hint: What is this surface?)*

4. *(20 pts)* A cylindrical aquarium is filled with water that has been stirred such that the flow field is $\vec{F}(x, y, z) = (-y, x, 0)$. A hemispherical net in the first octant has its boundary fixed as the circle in the xz-plane of radius 2 centered at $(3, 0, 0)$, and is oriented in the positive y-direction. Compute the flux of $\vec{F}$ through this net.

4

5. *(20 pts)* The downward oriented surface S is the part of the plane P (with equation $x+3y+2z = 18$) that lies directly above the rectangle in the xy-plane defined by $x \in [1,3]$ and $y \in [2,4]$. Compute the flux through S of the vector field $\vec{F}(x,y,z) = (1,2,3)$.