EXAM 2

Math 212, 2015-2016 Spring, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name ______________________________

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ____________________

1. __________

2. __________

3. __________

4. __________

5. __________

Total Score __________ (/100 points)

1. *(20 pts)* The barometric pressure P (measured in kPa) is dependent on position (x, y) (measured in miles east and north (respectively) of a fixed reference point) in a particular region as described by the equation below.

$$P = 101 + \frac{\sin(3x - 7y)}{6}$$

At a given moment, a car is located at $(0, 0)$ and moving at 60 miles per hour in the direction indicated by the vector $\langle 4, 3 \rangle$.

(a) Use the multivariable chain rule to compute $\frac{dP}{dt}$ as experienced by the car.

(b) Use the directional derivative to compute $\frac{dP}{dt}$ as experienced by the car.

2. *(20 pts)* The region R is bounded by the curves $y = -x^2$ and $y = x^2 - 8x$. The amount of corn yielded per unit area in R in a given season is given by $C(x, y) = x$. Compute the total quantity of corn yield by the entire region R.

3. *(20 pts)* The solid T is the region under the surface $8x + 4y + 2z = 24$ and above the triangle in the xy-plane with vertices at $(0, 1, 0)$, $(1, 2, 0)$, and $(2, 1, 0)$. The density is $\delta(x, y, z) = ke^{-z}$, and the mass is known to be M.

   Set up, but do not evaluate, a triple iterated integral representing the y coordinate of the centroid of T.

4. *(20 pts)* The solid R is bounded by the sphere of radius 2 centered at $(0, 0, 2)$ and the sphere of radius 1 centered at $(0, 0, 1)$. Set up, but do not evaluate, a triple iterated integral in the coordinate system of your choice that represents $\iiint_R f\, dV$.

5. *(20 pts)* A vector in the st-plane is rotated counterclockwise around the origin by the angle $\pi/4$ by the function

$$R(s,t) = \begin{pmatrix} (s-t)/\sqrt{2} \\ (s+t)/\sqrt{2} \end{pmatrix}$$

Use this function and the method of change of variables to compute the value of the integral $\iint_P x\,dx\,dy$, where P is the rectangle in the xy-plane with vertices at $(0,0)$, $(3\sqrt{2}, 3\sqrt{2})$, $(-\sqrt{2}, \sqrt{2})$, $(2\sqrt{2}, 4\sqrt{2})$.

(*You may NOT use a geometric argument to compute the Jacobian determinant.)*