EXAM 1

Math 212, 2015-2016 Fall, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name ______________________________

Disc.: Number__________ TA____________________ Day/Time____________________

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ____________________

1. __________

2. __________

3. __________

4. __________

5. __________

6. __________

Total Score __________ (/100 points)

1. *(20 pts)* Find a parametrization of the line L that is contained in the plane with equation $3x - 2y + z = 4$ and perpendicular to the line parametrized by $(x, y, z) = (2 + t, 3 - 2t, 1 + t)$.

2. *(10 pts)* The curve C is parametrized by

$$\vec{x}(t) = \begin{pmatrix} t-1 \\ t^2-t \\ t^3-t^2 \end{pmatrix}$$

At the moment this curve passes through the origin, compute the velocity, acceleration, and curvature.

3. *(10 pts)* Compute the limit below, or show that it does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{x^4-3xy^5}{x^4+y^4}$$

4. *(20 pts)* The surface Q has equation $-x^2 - y^2 + z^2 = 1$; the surface S is the part of Q above the xy-plane.

(a) Is Q a circular paraboloid, elliptical paraboloid, ellipsoid, hyperbolic paraboloid, hyperboloid of 1 sheet, hyperboloid of 2 sheets, hyperbolic cylinder, parabolic cylinder, or none of these? Explain your reasoning (do not simply cite a result from the book!).

(b) Is Q a level set of a function f? If so, indicate the domain, target, and formula for f.

(c) Is S the graph of a function g? If so, indicate the domain, target, and formula for g.

5. *(20 pts)* Suppose that z is a function of x and y, which themselves are functions of s and t as given by $x = 3s^2$ and $y = 2s - 4t^2$. Find a fully simplified expression for $\dfrac{\partial^2 z}{\partial s^2}$.

6. *(20 pts)* The concentration of air pollution (in units of ppm) in a region is given by

$$C(x, y) = \arctan(6 - x^2 - y^2)$$

Use the directional derivative to compute $\dfrac{dC}{ds}$ for a particle that is at the point $(1, 2)$ and moving with velocity $\langle 5, 12 \rangle$.