EXAM 2

Math 212, 2014-2015 Spring, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name ______________________________

Disc.: Number__________ TA____________________ Day/Time____________________

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ____________________

1. __________
2. __________
3. __________
4. __________
5. __________
6. __________

Total Score __________ (/100 points)

1. *(20 pts)* A particle is at the point $(2,0,1)$ and moving with velocity $\vec{v} = \langle 1,5,4\rangle$. The concentration of pollution in the air is a function of position by $C(x,y,z) = x^2 + xe^y - yz$.

   (a) How fast is C changing at the location of the particle as it moves?

   (b) What is the directional derivative of C at the point $(2,0,1)$ in the direction the particle is moving?

   (c) In what direction from the point $(2,0,1)$ is the function C increasing the fastest?

   (d) What is the maximum value of $D_{\vec{u}}C(2,0,1)$ among all unit vectors $\vec{u}$?

2. *(20 pts)* Compute the moment of inertia around the y-axis of the lamina in the xy-plane bounded by $y = 0$, $x = 1$, and $y = x^2$, whose density is given by $\delta(x, y) = y^2$.

3. *(20 pts)* The solid D is bounded by the three coordinate planes, the plane $x+y+z=2$, and the plane $y=1$. Find, but do not evaluate, a triple iterated integral representing $\iiint_D f(x,y,z)\,dx\,dy\,dz$.

4. *(20 pts)* Recall that a rotation by an angle θ counterclockwise around the origin is accomplished by the function

$$R(u, v) = (u \cos\theta - v \sin\theta, u \sin\theta + v \cos\theta)$$

Use this as a change of variables function to compute the integral $\iint_S x - y \, dx \, dy$, where S is the solid rectangle with corners at $(0, 0)$, $(5, 5)$, $(-2, 2)$, $(3, 7)$.

5. *(20 pts)* The region B is inside the sphere of radius 1 centered at $(0, -1, 0)$. Find, but do not evaluate, a triple iterated integral in spherical coordinates representing $\iiint_B y\,dx\,dy\,dz$