EXAM 1

Math 212, 2014-2015 Spring, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name Solutions

Disc.: Number__________ TA____________________ Day/Time____________________

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ____________________

1. __________

2. __________

3. __________

4. __________

5. __________

6. __________

Total Score __________ (/100 points)

1. *(20 pts)* The lines L_1 and L_2 are parametrized by $\vec{x} = (3-2t, 4t-1, 3t+2)$ and $\vec{x} = (6t+3, -1-t, 2-2t)$, respectively. Find the symmetric equations for the line that orthogonally intersects both L_1 and L_2.

L_1 and L_2 intersect at $(3,-1,2)$

Direction vectors are $\vec{v_1} = \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix}$, $\vec{v_2} = \begin{pmatrix} 6 \\ -1 \\ -2 \end{pmatrix}$.

Orthogonal to both of these is

$$\vec{w} = \vec{v_1} \times \vec{v_2} = \begin{pmatrix} -5 \\ 14 \\ -22 \end{pmatrix}$$

So the line is parametrized by

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} + t \begin{pmatrix} -5 \\ 14 \\ -22 \end{pmatrix}$$

Solving for t in each coordinate, we get

$$t = \boxed{\frac{x-3}{-5} = \frac{y+1}{14} = \frac{z-2}{-22}}$$

2. *(20 pts)* A plane is flying with position given by $\vec{x}(t) = (t^2 - 3t, 2t, 4t^4 + 5)$. At the moment when $t = 1$, what is the component of the planes velocity in the direction of the vector $(12, 3, 4)$?

$$\vec{V} = \vec{X}' = \begin{pmatrix} 2t-3 \\ 2 \\ 16t^3 \end{pmatrix}, \text{ so } \vec{v}(1) = \begin{pmatrix} -1 \\ 2 \\ 16 \end{pmatrix}$$

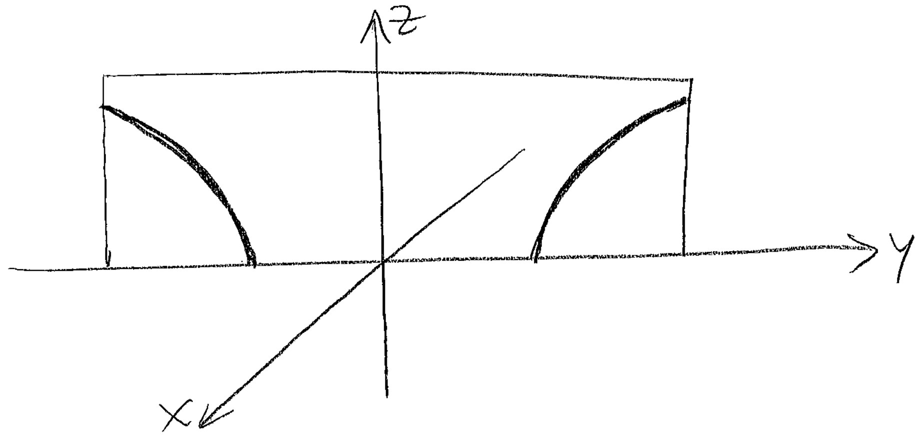
The component in direction of $\vec{W} = \begin{pmatrix} 12 \\ 3 \\ 4 \end{pmatrix}$ is

$$\text{comp}_{\vec{w}}(\vec{v}) = \frac{\vec{v} \cdot \vec{w}}{\|\vec{w}\|} = \frac{(-1)(12) + (2)(3) + (16)(4)}{\sqrt{12^2+3^2+4^2}}$$

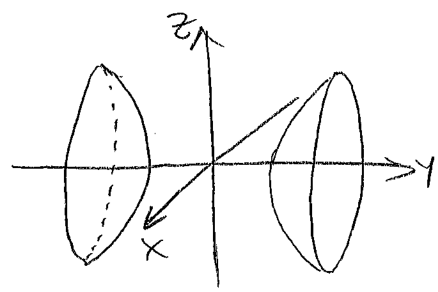
$$= \boxed{\frac{58}{13}}$$

3. *(20 pts)* The surface S has equation $4x^2 - y^2 + z^2 + 1 = 0$. Give a clear description of what S looks like, and explain how you come to that conclusion.

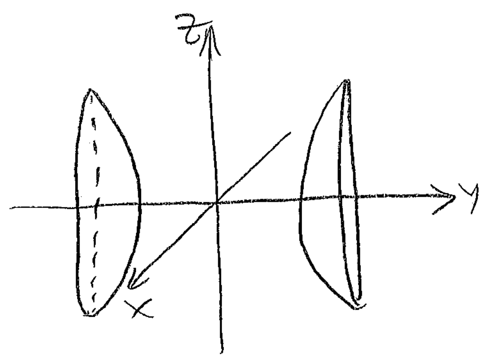
Begin with $x^2 - y^2 + z^2 + 1 = 0$, which is rot. symm. around y-axis; and in $\{x=0; z \geq 0\}$, this becomes the hyperbola $y^2 - z^2 = 1$ as below.



Rotating around y-axis we get $x^2 - y^2 + z^2 + 1 = 0$ is the equation for



Replacing "x" with "2x" in this equation gives the eqn
$$4x^2 - y^2 + z^2 + 1 = 0$$
for S which is the above "squished" in the x-direction



4. *(20 pts)* Find a function h such that one of the level sets of h is the graph of the function $f : \mathbb{R}^3 \to \mathbb{R}^1$ defined by $f(x, y, z) = 3xy - xz^2 + e^{xyz}$. What is the domain of h, and what is the target of h?

Graph of f has equation

$$W = 3xy - xz^2 + e^{xyz}$$

This is equivalent to

$$3xy - xz^2 + e^{xyz} - W = 0$$

which is the $h=0$ level set of $\boxed{h : \mathbb{R}^4 \to \mathbb{R}^1}$,

$$\boxed{h(x,y,z,w) = 3xy - xz^2 + e^{xyz} - W}$$

5. *(20 pts)* Find the function $G(x, y, z)$ that is the linear approximation to the function $p(x, y, z) = xy^4 + ye^{z^2}$ at the point $(3, 1, 0)$.

$$p(3,1,0) = 3\cdot 1 + 1\cdot e^0 = 4$$

$$\frac{\partial p}{\partial x} = y^4 \qquad \left.\frac{\partial p}{\partial x}\right|_{(3,1,0)} = 1$$

$$\frac{\partial p}{\partial y} = 4xy^3 + e^{z^2} \qquad \left.\frac{\partial p}{\partial y}\right|_{(3,1,0)} = 13$$

$$\frac{\partial p}{\partial z} = 2zye^{z^2} \qquad \left.\frac{\partial p}{\partial z}\right|_{(3,1,0)} = 0$$

$$G(x,y,z) = p(3,1,0) + \left(\left.\frac{\partial p}{\partial x}\right|_{(3,1,0)}\right)(x-3) + \left(\left.\frac{\partial p}{\partial y}\right|_{(3,1,0)}\right)(y-1) + \left(\left.\frac{\partial p}{\partial z}\right|_{(3,1,0)}\right)(z-0)$$

$$= 4 + 1(x-3) + 13(y-1) + 0(z-0)$$

$$= \boxed{x + 13y - 12}$$