EXAM 3

Math 212, 2014-2015 Fall, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name _____

"I have adhered to the Duke Community
Standard in completing this
examination."

1. _____

2. _____ Signature: _____

3. _____

4. _____

5. _____

6. _____

Total Score _____ (/100 points)

1. *(20 pts)* The curve C is parametrized by $\vec{x}(t) = (2t^2 - 4t, t + t^3, t^2)$, $0 \leq t \leq 1$, and the vector field $\vec{F}$ is given by $\vec{F}(\vec{x}) = \langle yze^{xy}, xze^{xy}, e^{xy} \rangle$. Compute $\int_C \vec{F} \cdot \vec{T}\, ds$.

2. *(20 pts)* The curve P is the intersection of the paraboloid $z = x^2 + y^2$ and the plane $z = 2x + 4y$, oriented clockwise as seen from above. The vector field $\vec{G}$ is given by $\vec{G}(\vec{x}) = \langle e^{x^2} + 3y + z, \sin(y^3) + x, 4y - 3z \rangle$. Compute $\int_P \vec{G} \cdot \vec{T}\, ds$.

3. *(20 pts)* The torus T has spherical equation $\rho = \sin\phi$ and is oriented inward, and the vector field $\vec{H}$ is given by $\vec{H}(\vec{x}) = \langle 2x - e^y, -y + 2yz, z - z^2 \rangle$. Compute $\iint_T \vec{H} \cdot \vec{n}\, dS$.

4. *(10 pts)* Perfectly balanced balloons are being pushed and spun through the air by the air flow described by the vector field $\vec{A}(\vec{x}) = \langle z - xz, 2x + yz, xy \rangle$.

   (a) For a balloon at the point $(1, 4, 1)$, find the velocity, and the axis of rotation of its spin.

   (b) What is the ratio of the spinning rates of balloons at $(1, 4, 1)$ and $(0, 0, 0)$?

5. *(10 pts)* Find the steady state current flow field $\vec{J}$ that generates the magnetic field $\vec{B} = \langle 0, -ze^{-y^2-z^2}, ye^{-y^2-z^2} \rangle$.

6. *(20 pts)* The surface H is the part of the hyperboloid of one sheet with equation $x^2 + y^2 - z^2 = 9$ that is between $z = -4$ and $z = 4$, oriented toward the z-axis. The vector field $\vec{F}$ is given by $\vec{F}(\vec{x}) = \langle x(x^2 + y^2), y(x^2 + y^2), -4z(x^2 + y^2)\rangle$. Compute $\iint_H \vec{F} \cdot \vec{n}\, dS$.