EXAM 1

Math 212, 2014-2015 Fall, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING. All answers must be simplified. All of the policies and guidelines on the class webpages are in effect on this exam.

Good luck!

Name Solutions

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ______________________

1. __________

2. __________

3. __________

4. __________

5. __________

6. __________

Total Score __________ (/100 points)

1. *(20 pts)*

(a) Use the geometric interpretations of cross product to find the vector $\vec{v}$ for which

i. $\vec{v} \times (3, 2, 1) = (1, 1, -5)$

ii. $\vec{v} \times (2, 3, 2) = (3, -2, 0)$ $\Longrightarrow \vec{v} \perp (1,1,-5)$ and $(3,-2,0)$

iii. $\|\vec{v}\| = \sqrt{14}$

So $\vec{v} \parallel$ to $(1,1,-5)\times(3,-2,0) = (-10,-15,-5)$

$= -5\,(2,3,1)$

and thus also $\parallel$ to $(2,3,1)$. Correcting for length,

$$\vec{v} = \pm\sqrt{14}\,\frac{(2,3,1)}{\|(2,3,1)\|} = \pm\,(2,3,1)$$

Checking for consistency with (i),(ii), we see $\boxed{\vec{v} = (2,3,1)}$.

(b) Compute the vector $\vec{w}$ whose component in the direction of $(3, 4)$ is 2, and whose component in the direction of $(5, 12)$ is 1.

$$\frac{\vec{w}\cdot(3,4)}{\|(3,4)\|} = 2 \quad \text{and} \quad \frac{\vec{w}\cdot(5,12)}{\|(5,12)\|} = 1$$

Writing $\vec{w} = (x,y)$, we have

$$3x + 4y = 10$$
$$5x + 12y = 13$$

Eliminating y,

$$4x = 17 \Rightarrow x = \frac{17}{4}$$

and then $y = \dfrac{10-3x}{4} = \dfrac{-11}{16}$

So $\boxed{\vec{w} = \left(\frac{17}{4}, \frac{-11}{16}\right)}$

2. *(15 pts)* The line L is parallel to the planes with equations $x - y - z = 0$ and $x + 2y + 3z = 9$, and passes through the point $(2, 3, 1)$. Find a parametrization of L, and the symmetric equations for L.

$\vec{v}$ is $\perp$ to $(1,-1,-1)$ and $(1,2,3)$, so

we choose $\vec{v} = (1,-1,-1) \times (1,2,3) = (-1,-4,3)$

With $\vec{x_0} = (2,3,1)$, the parametrization is

$$\boxed{\vec{x} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + t \begin{pmatrix} -1 \\ -4 \\ 3 \end{pmatrix}}$$

Solving for t gives the symmetric equations

$$t = \boxed{\frac{x-2}{-1} = \frac{y-3}{-4} = \frac{z-1}{3}}$$

3. *(15 pts)* Find the equation of the ellipsoid with center at $(4, -2, 3)$ that is tangent to all three of the coordinate planes.

Radii in x, y, z directions are $4, 2, 3$ (resp.).

$$x^2+y^2+z^2=1 \xrightarrow[\substack{\text{"}x\text{"} \rightsquigarrow \text{"}x/4\text{"} \\ \text{"}y\text{"} \rightsquigarrow \text{"}y/2\text{"} \\ \text{"}z\text{"} \rightsquigarrow \text{"}z/3\text{"}}]{} \frac{x^2}{16}+\frac{y^2}{4}+\frac{z^2}{9}=1$$

Moving the center to the required $(4,-2,3)$ gives

$$\xrightarrow[\substack{\text{"}x\text{"} \rightsquigarrow \text{"}x-4\text{"} \\ \text{"}y\text{"} \rightsquigarrow \text{"}y+2\text{"} \\ \text{"}z\text{"} \rightsquigarrow \text{"}z-3\text{"}}]{} \boxed{\frac{(x-4)^2}{16}+\frac{(y+2)^2}{4}+\frac{(z-3)^2}{9}=1}$$

4. *(15 pts)* The curve C in the xy-plane has equation $y = 4\sin(3x)$.

(a) Find the domain, target, and formula for a function f whose *graph* is the curve C.

$$\boxed{f : \mathbb{R}^1 \to \mathbb{R}^1, \quad f(x) = 4\sin 3x}$$

has graph $y = f(x) = 4\sin 3x$

(b) Find the domain, target, and formula for a function g for which one of the *level sets* is the curve C.

$$\boxed{g : \mathbb{R}^2 \to \mathbb{R}^1, \quad g(x,y) = y - 4\sin(3x)}$$

has a level set $0 = g(x,y) = y - 4\sin 3x$

$\Leftrightarrow \quad y = 4\sin 3x$

(c) Find the domain, target, and formula for a function h that *parametrizes* the curve C.

$$\boxed{h : \mathbb{R}^1 \to \mathbb{R}^2, \quad h(t) = (t, 4\sin 3t)} = (x,y)$$

follows the curve $y = 4\sin 3x$

5. *(15 pts)* Find the limit below, or show that it does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{x^4 + y^2}{x^2 + y^4}$$

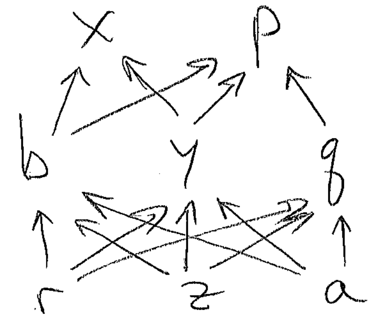
Along lines $y = mx$:

$$\lim_{x\to 0} \frac{x^4 + m^2x^2}{x^2 + m^4x^4} = \lim_{x\to 0} \frac{x^2 + m^2}{1 + m^4x^2} = m^2$$

This is different for different lines, so lim D.N.E.

6. *(20 pts)* Suppose we are given $x = by - q$, $b = rz + ra$, $y = r - z$, $p = b + yq$, and $q = arz$. Find an expression for $\partial p/\partial z$, and its value when $a = 2$, $r = 1$, $z = 3$.

p and z are related by three intermediate variables, b, y, and q. So



$$\frac{\partial p}{\partial z} = \frac{\partial p}{\partial b}\frac{\partial b}{\partial z} + \frac{\partial p}{\partial y}\frac{\partial y}{\partial z} + \frac{\partial p}{\partial q}\frac{\partial q}{\partial z}$$

$$= (1)(r) + (q)(-1) + (y)(ar)$$

$$= r - q + yar$$

When $a=2, r=1, z=3$, we also have $q=6$ and $y=-2$.

Then

$$\left.\frac{\partial p}{\partial z}\right|_{(2,1,3)} = 1 - (6) + (-2)(2)(1) = \boxed{-9}$$