

# EXAM 3

Math 212, 2013-2014 Spring, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING  
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

All answers must be simplified. All of the policies and guidelines  
on the class webpages are in effect on this exam.

Good luck!

Name \_\_\_\_\_

“I have adhered to the Duke Community  
Standard in completing this  
examination.”

1. \_\_\_\_\_

2. \_\_\_\_\_

3. \_\_\_\_\_

4. \_\_\_\_\_

5. \_\_\_\_\_

6. \_\_\_\_\_

Signature: \_\_\_\_\_

Total Score \_\_\_\_\_ (/100 points)

1. (15 pts) Compute  $\int_C \vec{F} \cdot \vec{T} ds$ , where  $\vec{F}(x, y) = (2x \sin(y^2), 2x^2 y \cos(y^2))$ , and the curve  $C$  is described by

$$\frac{x^2}{36} + \frac{y^2}{4} = 1 \quad \text{and} \quad y \geq 0, \text{ oriented to the right}$$

2. (15 pts) Compute  $\int_C \vec{F} \cdot \vec{T} ds$ , where  $\vec{F}(x, y) = (y \cos(xy) + y, x \cos(xy) - x)$ , and the curve  $C$  is the unit square oriented clockwise.

3. (15 pts) Compute the flux of the vector field  $\vec{F}(x, y, z) = (x + xy, x + xz, y + yz)$  through the inward oriented sphere of radius 4 centered at the origin.

4. (20 pts) The curve  $C$  is the regular hexagon in the plane  $x + y + z = 3$  with vertices at  $(2, 0, 1)$ ,  $(1, 0, 2)$ ,  $(0, 1, 2)$ ,  $(0, 2, 1)$ ,  $(1, 2, 0)$ ,  $(2, 1, 0)$ , oriented in the direction that these vertices are listed. Compute the circulation around this curve of the vector field

$$\vec{F}(x, y, z) = (xy^2z^2 + y, x^2yz^2 + z, x^2y^2z + x)$$

5. (15 pts) Compute the flux of the vector field  $\vec{F}(x, y, z) = (2, 2, 3)$  through the surface that is parametrized by  $\vec{x}(u, v) = (u + v, u - v, uv)$ ,  $0 \leq u \leq 1$ ,  $0 \leq v \leq 1$ .

6. (20 pts) The surface  $S$  is described in cylindrical coordinates by  $r = 1 + z^2$ ,  $-1 \leq z \leq 1$ , and is oriented outward. Compute the flux through  $S$  of the vector field

$$\vec{F}(x, y, z) = ((x^2 + y^2 - 4)x, (x^2 + y^2 - 4)y, -4z(x^2 + y^2 - 2))$$