EXAM 1

Math 212, 2013-2014 Spring, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name ______________________________

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ______________________

1. __________

2. __________

3. __________

4. __________

5. __________

6. __________

7. __________

Total Score __________ (/100 points)

1. *(10 pts)* For the vectors below, find their magnitudes and the cosine of the angle between them.

$$\vec{v} = \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix} \quad \text{and} \quad \vec{w} = \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix}$$

2. *(15 pts)* Find the distance between the point $\vec{p} = (3, -2, 5)$ and the plane with equation $2x - 3y + 6z = 4$. *(Hint: How does this relate to the $\vec{n}$ component of the vector $\vec{p} - \vec{x}_0$, where $\vec{x}_0$ is in the plane and $\vec{n}$ is normal to the plane?)*

3. *(15 pts)* Consider the vectors

$$\vec{u} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad \text{and} \quad \vec{v} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} \quad \text{and} \quad \vec{w} = \begin{pmatrix} -1 \\ -2 \\ 5 \end{pmatrix}$$

(a) Find the area of the parallelogram defined by $\vec{u}$ and $\vec{v}$.

(b) Find the volume of the parallelepiped defined by $\vec{u}$, $\vec{v}$, and $\vec{w}$, and determine if the listing $\vec{u}$, $\vec{w}$, $\vec{v}$ *(observe carefully!)* is in right hand order or left hand order.

(c) Use a clever application of the equation below to show that for all vectors $\vec{x}, \vec{y} \in \mathbb{R}^3$ the listing $\vec{x} \times \vec{y}$, $\vec{x}$, $\vec{y}$ is never in left hand order. *Be sure to explain your reasoning.*

$$\det \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix} = \vec{a} \cdot \left(\vec{b} \times \vec{c} \right)$$

4. *(15 pts)* Compute the velocity, acceleration, and curvature of the parametric curve $\vec{x}(t) = (t^2 + 1, 2t - 1, t^3 - 5)$ at the point $(2, 1, -4)$.

5. *(15 pts)*

(a) The curve C in the xy-plane has equation $y = x^2 - 5$. Find the equation of the surface S_1 obtained by rotating C around the y-axis in xyz-space.

(b) The hyperboloid H has equation $x^2 - 3y^2 + 5z^2 = 1$. Find the equation of the surface S_2 obtained by stretching H by a factor of 4 in the x-direction.

(c) Find the equation of the surface S_3 obtained by translating H (from the previous part) by the vector $\vec{v} = (1, 3, 6)$ and then reflecting the result through the xz-plane.

6. *(15 pts)* In this question we consider the function $f : \mathbb{R}^2 \to \mathbb{R}^1$ defined by $f(x, y) = (y - x^2)^2$.

(a) Find the equation of the level set of this function that includes the point $(1, 2)$.

(b) Explain how you know the level set from the previous part is not the graph of any other function.

(c) Find the only function g whose graph is also a level set of f.

7. *(15 pts)* Compute (or show it does not exist) the limit below.

$$\lim_{\vec{x} \to \vec{0}} \frac{x^2}{x^2 + y^2}$$