EXAM 1

Math 103, Spring 2008-2009, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

Good luck!

Name ____________________________________

ID number________________________

1. ____________
2. ____________
3. ____________
4. ____________
5. ____________
6. ____________
7. ____________
8. ____________
9. ____________

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ________________________

Total Score ____________ (/100 points)

Consider the vectors

$$\vec{u} = \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix} \qquad \vec{v} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \qquad \vec{w} = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$$

1. *(12 pts)* What is the volume of the parallelepiped that is defined by these three vectors?

2. *(10 pts)* Is the ordered list of vectors $\vec{u}, \vec{v}, \vec{w}$ in right hand order, or not in right hand order? Make sure you explain your reasoning.

Suppose we are given a point $\vec{s}$, and a plane P defined by the equation $\vec{n}\cdot\vec{x} = \vec{n}\cdot\vec{x}_0$, given a normal vector $\vec{n}$ (pointing to the same side of P that the point $\vec{s}$ is on) and a point $\vec{x}_0$ in P.

On this page you will derive the formula for the distance from $\vec{s}$ to P (in terms of the givens $\vec{n}$, $\vec{s}$, and $\vec{x}_0$) by considering the point $\vec{r}$ on P that is closest to $\vec{s}$, and the difference vector $\vec{d} = \vec{s} - \vec{r}$. (You should draw a figure representing these objects.)

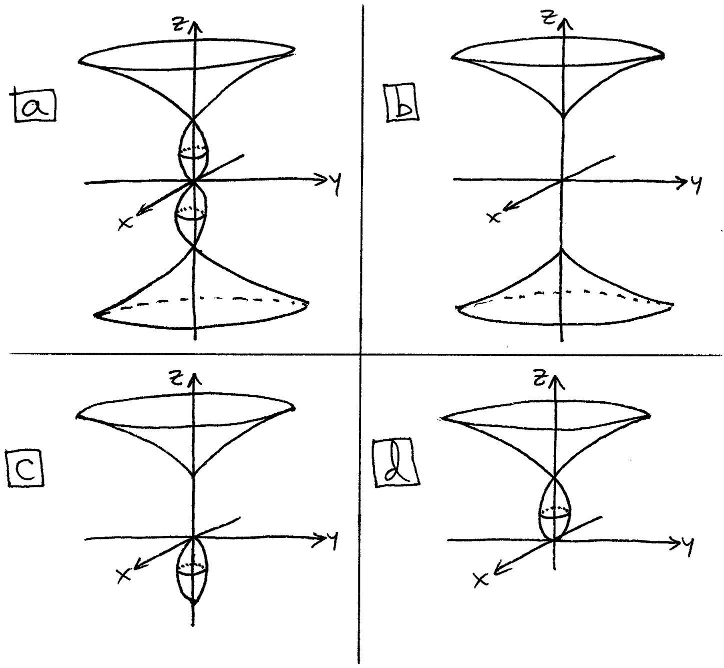
3. *(10 pts)* Use a geometric argument to compute $\vec{n}\cdot\vec{d}$ in terms of $\|\vec{n}\|$ and $\|\vec{d}\|$. (Make sure to explain all of the important points in your reasoning!)

4. *(10 pts)* Use the above definition of $\vec{d}$ to compute $\vec{n}\cdot\vec{d}$ in terms of dot products of $\vec{n}$, $\vec{s}$, and $\vec{x}_0$, and then combine with the above result to solve for the desired distance as $\|\vec{d}\|$ in terms of the givens.

Consider the parametric curve $\vec{x}(t)$ for which we know the velocity is given by $\vec{v}(t) = (2t, 6t^2, \pi\sin(\pi t))$.

5. *(12 pts)* Suppose that we know $\vec{x}(0) = \vec{0}$. Compute $\vec{x}(1)$.

6. *(12 pts)* Find the curvature of this path at the point where $t = 1$.



7. *(12 pts)* Which one of the surfaces above (a, b, c, or d) is the most accurate representation of the solution set to the equation

$$\sqrt{x^2+y^2} = z^3 - z$$

(Make sure to explain your reasoning!)

8. *(12 pts)* Compute the following limit:

$$\lim_{\vec{x}\to\vec{0}} \frac{x^2y^2 - xy^3}{x^4 + y^4}$$

9. *(10 pts)* Find a function g such that one of the level sets of g is the graph of the function $f : \mathbb{R}^2 \to \mathbb{R}^1$, given by

$$f(x, y) = \sin x - e^{xy}$$