EXAM 3

Math 103, Spring 2007-2008, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.
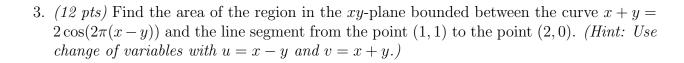
YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

Good luck!

Name _____

ID number_____

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"I have adhered to the Duke Community
Standard in completing this
examination."

Signature: _____

Total Score _____ (/100 points)

1. *(10 pts)* Find the moment of inertia around the x-axis of the solid bounded by the planes $z = 0$, $x + y = 1$, $y = 1 + x$, and $y = z$, with constant density δ.

2. *(12 pts)* Find the volume of the solid bounded by the surface with spherical equation $\rho = 1 - \cos\phi$.

3. *(12 pts)* Find the area of the region in the xy-plane bounded between the curve $x + y = 2\cos(2\pi(x-y))$ and the line segment from the point $(1,1)$ to the point $(2,0)$. *(Hint: Use change of variables with $u = x - y$ and $v = x + y$.)*

4. *(12 pts)* Compute the line integral $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = (x^3 - z, y^3, y + z^3)$ and C is parametrized by $\vec{r}(t) = (\cos^4 t, \sin^3 t, 7)$ with $t \in [0, 2\pi]$.

5. *(10 pts)* Compute the line integral $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = (ze^{xz} + e^x, 2yz, xe^{xz} + y^2)$ and C is parametrized by $\vec{r}(t) = (e^{t^2}, e^{t^3}, t^4)$ with $t \in [0, 1]$.

6. *(12 pts)* Compute the line integral $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = (y + \sin x, x^2 + e^y)$ and C is the boundary of the region between the curves $y = x^4$ and $y = 32 - x^4$.

7. *(10 pts)* Compute the line integral $\int_C \vec{F} \cdot d\vec{r}$, where C is parametrized by $\vec{r}(t) = (t + t(t-1)e^t, (t-1)\sin t)$ with $t \in [0,1]$, and the field $\vec{F} = (P, Q)$ is known only to satisfy the equations

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 0$$
$$P(x, 0) = 3x^2$$
$$Q(x, 0) = x^3$$

8. *(12 pts)* Compute the flux given by $\iint_S \vec{F} \cdot \vec{n} \, dS$, where $\vec{F} = (y, -x, z)$ and the surface S is defined by $z = 0$, $0 \le \theta \le \pi$ and $1 \le x^2 + y^2 \le 4$.

9. *(10 pts)* Compute the flux given by $\iint_S \vec{F} \cdot \vec{n} \, dS$, where $\vec{F} = (y^3 + z^2, xy - xz^2, xe^y)$ and the surface S is the boundary of the solid defined by $x, y, z \ge 0$ and $x + y + z \le 1$.