EXAM 2

Math 103, Spring 2007-2008, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

Good luck!

Name ______________________________

ID number____________________

1. ____________
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"I have adhered to the Duke Community Standard in completing this examination."

Signature: ____________________

Total Score ____________ (/100 points)

1. *(12 pts)* If the following limit exists, compute its value. If it does not exist, show how you know that it does not.

$$\lim_{(x,y)\to(0,0)} \frac{x^3y - xy^3}{x^4 + y^4}$$

2. *(12 pts)* Use the total derivative to estimate the value of the function

$$f(x, y, z) = x + e^{(3y+z)} \ln(x - z)$$

at the point $(x, y, z) = (1.01, .03, -.02)$.

3. *(14 pts)* Suppose that $w = w(x, y, z)$ has continuous second derivatives. Use the chain rule to compute the partial derivatives $\frac{\partial w}{\partial s}$ and $\frac{\partial^2 w}{\partial s \partial t}$, both when $s = 1$ and $t = 2$, given the relationships $x = 2st - t^2$, $y = t^2 - 3s$, $z = t^2$. Your answer will be written in terms of the partial derivatives of w with respect to x, y, and z. *(DO NOT compute the composition explicitly to answer this question.)*

4. *(12 pts)* What is the rate of change of the value of the function $f(x, y, z) = e^{(x+2y)} - (yz^2)$ with respect to distance traveled, when at the point $(3, 0, 2)$ and moving in the direction indicated by the vector $(1, 3, 2)$?

5. *(14 pts)* Find the absolute maximum value of the function $f(x, y, z) = x^2 - y^2 + z^2$ on the solid unit ball centered at the origin.

6. *(12 pts)* Consider the function $f(x, y) = x^3y - xy^3$. Confirm that f has a critical point at the origin, and classify that critical point as being a local maximum, local minimum, or saddle point.

7. *(12 pts)* Compute the volume under the surface $z = x+3$, above the region in the xy-plane bounded by the curves $y = x^3 - x$, $y = -6$ and $x = 1$.

8. *(12 pts)* Find the volume that is bounded between the graphs of the functions $f(x, y) = x^2 + e^{xy}$ and $g(x, y) = e^{xy} - y^2 + 1$. (Leave your answer as a single variable integral in terms of x.)