EXAM 3

Math 103, Fall 2006-2007, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

Good luck!

Name ______________________________________

ID number_________________________

1a. ____________ (/10 points)

1b. ____________ (/15 points)

1c. ____________ (/15 points)

1d. ____________ (/15 points)

2a. ____________ (/15 points)

2b. ____________ (/15 points)

2c. ____________ (/15 points)

Total ____________ (/100 points)

"I have adhered to the Duke Community Standard in completing this examination."

Signature: _________________________

1. For each of the following fields $\vec{F}$ and paths C, compute the line integral

$$\int_C \vec{F} \cdot \vec{T}\, ds$$

(a) $\vec{F} = \left\langle 2xy + y^2, 2xy + x^2 \right\rangle$, C is parametrized by $\vec{r}(t) = \left(e^{t^2-t}, t^3 + t^2\right)$, $t \in [0, 1]$.

(b) $\vec{F} = \left\langle e^x + y, -x + e^{y^2} \right\rangle$, C is the boundary of the set R, which is the union of the solid unit square $[0, 1] \times [0, 1]$ and the solid unit disk.

(c) $\vec{F} = \left\langle x^2 e^x, x + y^2, y + z^3 \right\rangle$, C is parametrized by $\vec{r}(t) = \left(\cos t, \sin t, \sin t + \cos t \right)$, $t \in [0, 2\pi]$. *(Hint: Notice that the curve C is entirely in the plane $x + y - z = 0$.)*

(d) $\vec{F} = \left\langle (x^3y^4z^3+3x^2y^3z^3)e^{xy}, (x^4y^3z^3+3x^3y^2z^3)e^{xy}, 3x^3y^3z^2e^{xy}+1 \right\rangle$, C is parametrized by $\vec{r}(t) = \Big(-\sin t, \cos t, t\Big)$, $t \in [0, 2\pi]$. *(Hint: You may use the fact that* $\nabla \times \vec{F} = \vec{0}$*.)*

2. For each of the following fields $\vec{F}$ and surfaces S, compute the surface integral

$$\iint_S \vec{F} \cdot \vec{n}\, dS$$

(a) $\vec{F} = \left\langle x, z, y \right\rangle$, S is parametrized by $\vec{r}(u, v) = \left(uv, u+v, u-v\right)$, $u \in [0, 1]$, $v \in [0, 1]$.

(b) $\vec{F} = \left(x^2+y^2z, z^2e^x, x^3\right)$, S is the boundary of the solid $R = \{x^2+y^2+z^2 \leq 1, z \geq 0\}$.

(c) $\vec{F} = \left(ye^{z^2}, x^3z^2, 2\right)$, $S = \left\{x^2 + y^2 + (z+4)^2 = 25, z \geq 0\right\}$ oriented downward.