EXAM 1

Math 103, Fall 2006-2007, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

Good luck!

Name ____________________________________

ID number________________________

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ________________________

Total Score: ____________ (/100 points)

1. *(20 points)* Points: ____________

(a) Suppose f is a function of x, y and z, with $x = u^2 + v$, $y = u + v^2$, and $z = uv$. Suppose further that

$$\frac{\partial f}{\partial u}(0,1) = 2 \quad \text{and} \quad \frac{\partial f}{\partial z}(1,1,0) = 5$$

Compute $\frac{\partial f}{\partial y}$(1,1,0).

(b) The density of mosquitos (measured in thousand per cubic meter) in the vicinity of a swamp is given by $f(x,y) = \frac{1}{(3x^2+4y^2)}$, where x and y are measured in miles. Bob is located at $(1,1)$; in what (unit vector) direction should he run so that the density of mosquitos is decreasing the fastest? And if his top running speed is ten miles per hour, at what rate is the density decreasing as he runs in the optimum direction?

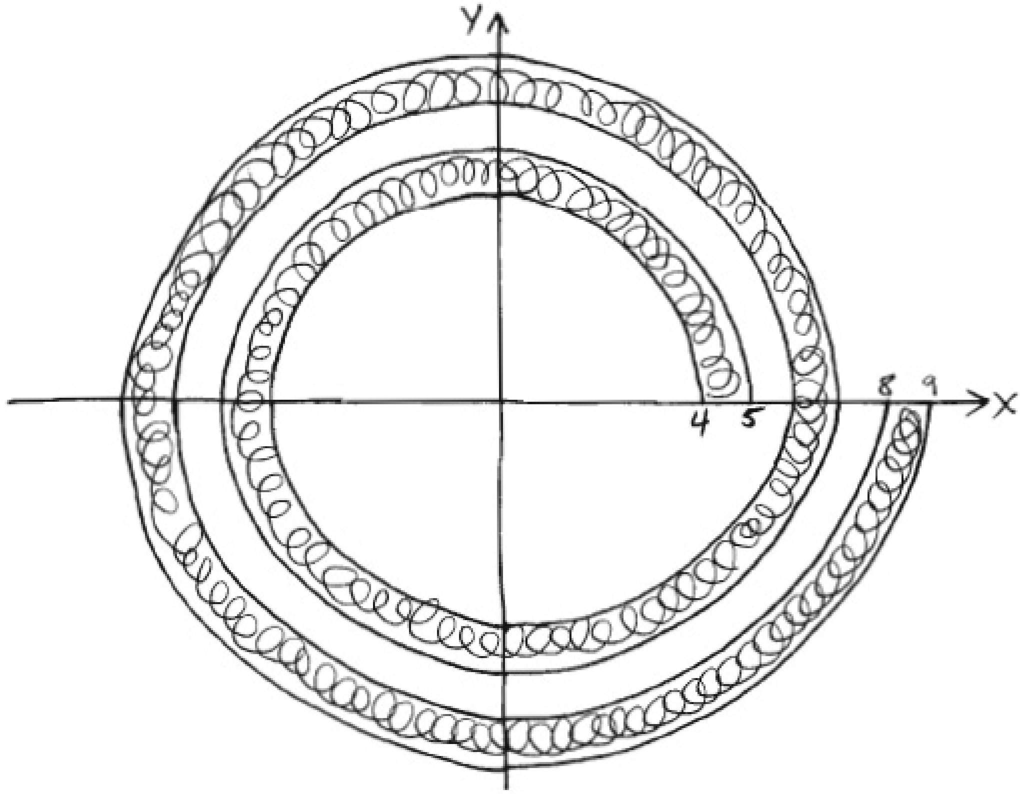
2. *(20 points)* Points: ____________

(a) Find the coordinates of the point (above the x-axis) on the ellipse $4x^2 + 9y^2 = 36$ that is closest to the point $(1, 0)$. *(Hint: Minimize the square of the distance.)*

(b) The function $f(x, y) = e^x + e^{x+y} - 2x - y$ has a critical point at the origin. Determine if this critical point is a local minimum, local maximum, or saddle point.

3. *(20 points)* Points: ____________

(a) The population density in the xy-plane of bacteria at a point (x, y) is given by the function $\delta(x, y) = x + 2y$, and the food availability is given by the function $f(x, y) = 2x - y$. Compute the total population of bacteria over the domain where the population density is between 1 and 3 and the food availability is between 1 and 5.



(b) The figure above shows the curves represented by the polar equations $r = \frac{\theta}{\pi}$ and $r = 1 + \frac{\theta}{\pi}$, where θ is allowed to range between 4π and 8π. Compute the area of the shaded region.

4. *(20 points)* Points: ____________

Each of the following two quantities can be computed without directly computing an iterated integral.

(a) Compute the volume under the surface defined by the equation $z = 1 - \cos r$ over the disk in the xy-plane with radius 2π centered at the origin.

(b) Compute the integral

$$\iint_D \frac{x^5 - y^5}{x^2 + y^2 + 1}\, dx\, dy$$

where D is the region bounded by the curves $y = x^2$ and $x = y^2$.

5. *(20 points)* Points: ____________

For each of the following, completely set up iterated integrals representing the described quantities, but DO NOT EVALUATE those integrals.

(a) The moment of inertia around the x-axis of the domain bounded by the surfaces $x^2 + z^2 = 1$, $y = 0$, $y = 2 - x - z$, with density given by $\delta(x, y, z) = x^2$.

(b) The x-coordinate of the centroid of the domain bounded by $(x-1)^2+y^2=1$, $z=0$, $z=x^2+y^2$, with density given by $\delta=x^2+y^2$ (you must use cylindrical coordinates for this integral).

(c) The z-coordinate of the centroid of the region R inside the sphere of radius 1 centered at $(0, 1, 0)$ and above the cone $z = r$ (you must use spherical coordinates for this integral).