

EXAM 1

Math 103, Fall 2006-2007, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

Good luck!

Name _____

ID number _____

“I have adhered to the Duke Community Standard in completing this examination.”

Signature: _____

Total Score: _____ (/100 points)

1. (20 points)

Points: _____

- (a) Compute the cosine of the angle between the vectors \vec{x} and \vec{y} , which both have tails at the point $(3, 2, 5)$ and have heads at the points $(1, 3, 6)$ and $(4, 3, 1)$, respectively.

- (b) The two vectors \vec{v} and \vec{w} are parallel to the plane with equation $3x - 4y + 12z = 7$, they define a parallelogram with area 26, and $\vec{v} \times \vec{w}$ has negative z -coordinate. Compute $\vec{v} \times \vec{w}$.

2. (20 points)

Points: _____

- (a) The curve C is defined by the equation $\left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$. Compute the curvature of this curve at the point $(3, 0)$. (*Hint: Parametrize this curve by starting with the parametrization $(\cos t, \sin t)$ of the unit circle, and then note that an ellipse is obtained from that by stretching x and y by factors.*)
- (b) What kind of curves are the horizontal cross-sections (traces) of the surface described by the equation $x^2(z^2+1)^2 + xz^2 - y^2e^z = e^z$? (*Make sure to explain your reasoning!*)

3. (20 points)

Points: _____

Compute the following limits, or show that they do not exist:

(a)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2y + y^2e^x}{x^2 + e^y}$$

(b)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{(x^2 - y^2)(x + y)}{x^2 + y^2}$$

(c)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^3y}{x^6 + y^2}$$

4. (20 points)

Points: _____

- (a) Consider the graph of $f(x, y)$, and the tangent plane to this graph above the fixed point (a, b) in the domain. Use the fact that the tangent plane must pass through the point $(a, b, f(a, b))$ to show, as was claimed in class, that the equation of the tangent plane can be put into the form

$$z = f(a, b) + P(x - a) + Q(y - b)$$

for some constants P and Q .

- (b) Use the geometric properties of the tangent plane to derive the values for these constants P and Q . Make sure to explain your reasoning.

5. (20 points)

Points: _____

The domain D is the solid triangle in \mathbb{R}^2 with vertices $(0,0)$, $(1,0)$, $(0,1)$. Find the absolute maximum value of the function $f(x,y) = x^2y$ over the domain D .