# EXAM 2

Math 103, Summer 2006, Clark Bray.

You have 75 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

Good luck!

Name _____

ID number_____

1. _____ (/20 points)

2. _____ (/20 points)

3. _____ (/20 points)

4. _____ (/20 points)

5. _____ (/20 points)

"I have adhered to the Duke Community Standard in completing this examination."

Signature: _____

Total _____ (/100 points)

1. Suppose we know that $s$ and $t$ are functions of $x$ and $y$, and $f = s^3 - 3st^2 + 6t^2$.

   (a) For certain specific values of $x$ and $y$ we have $s = 1$ and $t = 3$, $\frac{\partial f}{\partial y} = 3$ and $\frac{\partial t}{\partial y} = 2$. Use the chain rule to compute $\frac{\partial s}{\partial y}$.

(b) Locate and classify all critical points of $f$ thought of as a function of $s$ and $t$.

2. The Gotham City Police Department has been warned that the Joker will be attempting to rob one of three banks in the Greater Gotham area tonight. Given the clues, Commissioner Gordon thinks that there is a 60% chance that the First National Bank will be hit, a 10% chance for the Second National Bank, and a 30% chance for the Third National Bank.

   For whichever bank is hit, the utility (in terms of the likelihood that they will be able to catch the Joker and protect the bank) of having $x$ police stationed at that bank is estimated to be $lnx$. So if police are stationed at the three banks above in numbers $x_1$, $x_2$, $x_3$, respectively, the expected total utility is

$$U(x_1, x_2, x_3) = (.6)\ln(x_1) + (.1)\ln(x_2) + (.3)\ln(x_3)$$

   The GCPD has five hundred police available for Gordon to assign to protect these three banks, but he does not know how he should distribute them among the three banks in order to achieve maximum expected utility. As usual he consults Batman – but Batman never took Math 103, and so Batman consults you.

   How many police should you instruct Batman to post at each of the three banks? Make sure to explain your reasoning clearly.

3. Compute the following, subject to the given conditions on your methods. Make sure to explain clearly all of your reasoning.

   (a) The integral of the function $f(x, y) = x^3 - y^3 + (x - y)x^4y^4$ over the domain which is the intersection of the disks represented by $(x - 1)^2 + (y + 1)^2 \leq 9$ and $(x + 1)^2 + (y - 1)^2 \leq 9$. Use any techniques from this course.

   (b) The $y$-coordinate of the centroid of the upper half of the unit disk, using Pappus' theorem and your knowledge of the volume of the unit sphere and the area of the unit disk.

(c) The integral below, where $D$ is the part of the unit disk in the fourth quadrant, using any techniques from this course.

$$\iint_D y^3 \, dA$$

4. Set up these problems as iterated integrals, *but do not evaluate*:

   (a) Find the mass of the region D bounded by the surfaces $y^2+z^2=1$, $x+y+z+3=0$ and $y^2+z^2+1=x$, where the density is given by the function $\delta(x,y,z)=xy^2e^z$.

   (b) Rewrite the following integral with the order of the differentials switched.

$$\int_{-2}^{3}\int_{-8}^{x^3} f(x,y)\,dydx$$

(c) Find the moment of inertia around the $y$-axis of the lamina in the $xz$-plane defined by $(x-3)^2+(z-2)^2=1$, with density given by $\delta(x,z)=e^{x+z}$.

5. The domain $D$ in the $xy$-plane is the image of the unit square in the $uv$-plane by the function $T(u,v) = ((u+1)^2 - v^2, 2(u+1)v)$. Compute the integral

$$\iint_D \frac{1}{\sqrt{x^2+y^2}}\,dx\,dy$$

*(Hint: The following algebra might be convenient:)*

$$\left[(u+1)^2 + v^2\right]^2 = \left[(u+1)^2 - v^2\right]^2 + \left[2(u+1)v\right]^2$$