# EXAM 1

Math 103, Summer 2006, Clark Bray.

You have 75 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

Good luck!

Name ____________________________________

ID number________________________

1. ____________ (/20 points)

2. ____________ (/20 points)

3. ____________ (/20 points)

4. ____________ (/20 points)

5. ____________ (/20 points)

Total ____________ (/100 points)

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ________________________

1. Consider the following three points: $\vec{P} = (2, 3, 4)$, $\vec{Q} = (4, -7, 1)$, $\vec{R} = (-1, -1, 4)$.

   (a) Compute the distance from $\vec{P}$ to $\vec{Q}$.

   (b) Compute the angle between the two line segments attaching the point $\vec{R}$ to the points $\vec{P}$ and $\vec{Q}$.

(c) Compute the area of the triangle whose vertices are at the points $\vec{P}$, $\vec{Q}$ and $\vec{R}$.

2. Find the equation for the plane that is parallel to and equidistant from each of the following two parametric lines:

$$\begin{bmatrix} 4t \\ t \\ -t \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 3-t \\ 2+2t \\ 3+3t \end{bmatrix}$$

3. Suppose that a particle begins at rest at $t = 0$, and that the acceleration as a function of time is given by

$$\vec{a}(t) = \begin{bmatrix} 6t+1 \\ 3t^2 \\ 4t \end{bmatrix}$$

Find the curvature of the path that this particle follows, at time $t = 0$.

4. Compute each of the following limits, if it exists.

   (a) $\lim_{\vec{x}\to(1,2,3)} \frac{(x+y-z)^3}{(x^2+y^2+z^2)}$

   (b) $\lim_{\vec{x}\to\vec{0}} \frac{x^3y-y^3x}{x^2+y^2}$

(c) $\lim_{\vec{x}\to\vec{0}} \frac{x^2-xy}{x^2+y^2}$

5. Consider the surface $S$ that is the graph of the function $f(x, y) = x^2 + y^2$. In this problem we will find an alternative derivation for the equation of the tangent plane to the surface, at the specific point $(1, 2, 5)$.

   (a) Let $L_1$ be the line that is tangent to this surface at the point $(1, 2, 5)$ and whose projection to the $xy$-plane is parallel to the $x$-axis. Similarly, let $L_2$ be the line that is tangent to this surface at the point $(1, 2, 5)$ and whose projection to the $xy$-plane is parallel to the $y$-axis.

   Let the direction vectors $\vec{v}_1$ and $\vec{v}_2$ for these lines be given by $(1, 0, a)$ and $(0, 1, b)$, respectively. Use your knowledge of partial derivatives to compute the values for $a$ and $b$. (Make sure to explain your reasoning clearly.)

(b) Use the fact that $\vec{v}_1$ and $\vec{v}_2$ are tangent to the surface to find a normal vector $\vec{n}$ for the tangent plane, and then write down the equation of the tangent plane. (Make sure to explain your reasoning clearly.)