

EXAM 3

Math 103, Spring 2006, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

Good luck!

Name _____

ID number _____

1. _____ (/30 points)

2. _____ (/20 points)

“I have adhered to the Duke Community
Standard in completing this
examination.”

3. _____ (/40 points)

Signature: _____

4. _____ (/10 points)

Total _____ (/100 points)

1. For each of the following fields $\vec{F} = (P, Q)$ and paths C , compute the line integral

$$\int_C \vec{F} \cdot \vec{T} \, ds = \int_C P \, dx + Q \, dy$$

- (a) $\vec{F} = (y + 1, -x)$, $C = \{x^2 + y^2 = 4, \quad y \geq 0\}$ oriented to the left.

- (b) $\vec{F} = (y+1, -x)$, C is the polygonal path with vertices (in order) $(0, 0)$, $(0, 3)$, $(2, 1)$, $(4, 3)$, $(4, 0)$, $(0, 0)$.

- (c) $\vec{F} = \left(3 + (2x^2y^3 + y)e^{x^2y^2}, (2x^3y^2 + x)e^{x^2y^2} \right)$, C is the upper half of the unit circle, oriented to the right. (*Hint: Is this field path-independent? How might that help?*)

2. For each of the following fields $\vec{F} = (P, Q, R)$ and paths C , compute the line integral

$$\int_C \vec{F} \cdot \vec{T} \, ds = \int_C P \, dx + Q \, dy + R \, dz$$

- (a) $\vec{F} = (yz, xz, xy)$, C is parametrized by $\vec{r}(t) = (t(t+2), t^2, e^{t^3})$, $t \in [0, 1]$.

(b) $\vec{F} = (e^x, x^2 e^y, y + x^2 z^3)$, C is parametrized by $\vec{r}(t) = (2, \cos(t), \sin(t))$, $t \in [0, 2\pi]$.

3. For each of the following fields \vec{F} and surfaces S , compute the surface integral

$$\iint_S \vec{F} \cdot \vec{n} \, dS$$

(a) $\vec{F} = (x, y, z)$, S is parametrized by $\vec{r}(u, v) = (u^2, v^2, u + v)$, $u \in [0, 1]$, $v \in [0, 1]$.

(b) $\vec{F} = \nabla \times (0, 0, e^{\sin(x^3y)})$, $S = \{z = 36 - x^2 - 9y^2, z \geq 0\}$ oriented downward.

(c) $\vec{F} = (y^2z, y + xe^z, x^2y^3)$, S is the unit cube, oriented inward.

(d) $\vec{F} = (y^2z, xez, x^2y^3)$, $S = \{z = 36 - x^2 - 9y^2, z \geq 0\}$ oriented downward.

4. Show that the volume of a solid region R in xyz -space, whose boundary is the surface ∂R , can be computed with the formula

$$\text{volume}(R) = \iiint_{\partial R} x \, dydz$$