EXAM 2

Math 103, Spring 2006, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

Good luck!

Name ______________________________________

ID number_________________________

1. _____________ (/20 points)

2. _____________ (/15 points)

3. _____________ (/15 points)

4. _____________ (/20 points)

5. _____________ (/30 points)

"I have adhered to the Duke Community Standard in completing this examination."

Signature: _________________________

Total _____________ (/100 points)

1. (a) The surface S is described by the equation $x^2 + xy + 2y^2 + 3z^2 = 7$. Use the Implicit Function Theorem to find $\frac{\partial z}{\partial y}$ at the point $(1, 1, 1)$. (Make sure to explain why you can use the IFT!)

(b) Find and classify (as local maxima, local minima, or saddle points) all critical points of the function $f(x, y) = 4xy - 2x^4 - y^2$.

2. Use Lagrange multipliers to find the point on the surface defined by $x^2 + 2y^2 + 3z^2 = 1$ that maximizes the value of $x^3 + y^3 + z^3$.

3. Compute the volume of the solid bounded by the surfaces $y = -1$, $y = x + 1$, $y = 1 - x$, $z = 0$, and $z = x^2 + y^2$.

4. The region D in the xy-plane is bounded by the four lines with equations $2x + y = 1$, $2x + y = 3$, $y - 2x = 2$, $y - 2x = 4$. Find a convenient change of variables function (using new variables u and v), transform the integral below to an integral over a rectangle in the new coordinate plane, and then evaluate that integral.

$$\iint_D (16x^2 - 4y^2)\, dx\, dy$$

5. Set up the following, but <u>do not evaluate</u>.

   (a) A single, iterated triple integral in cylindrical coordinates representing the z coordinate of the centroid of the mass inside the unit sphere, above the xy-plane, and below the plane $z = 1/2$, with density $\delta = x^2 + z^4$.

(b) A single, iterated double integral representing the moment of inertia around the x-axis of the lamina which is the part of the unit disk above the graph of $y = |x|$, with density given by $\delta = e^{xy}$. *(Hint: Choose an appropriate coordinate system for your integral.)*

(c) A single, iterated triple integral representing the mass of the region contained inside the sphere of radius a with center at $(0, 0, a)$, and below the cone $z = r$. *(Hint: Choose an appropriate coordinate system for your integral.)*