## EXAM 2

Math 103, Spring 2006, Clark Bray.

You have 50 minutes.

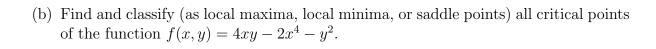
No notes, no books, no calculators.

## YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

Good luck!

Name		
	ID number	
1	(/20 points)	"I have adhered to the Duke Community Standard in completing this examination."  Signature:
2	(/15 points)	
3	(/15 points)	
4	(/20 points)	
5	(/30 points)	
Гotal	(/100 points)	

1. (a) The surface S is described by the equation  $x^2 + xy + 2y^2 + 3z^2 = 7$ . Use the Implicit Function Theorem to find  $\frac{\partial z}{\partial y}$  at the point (1,1,1). (Make sure to explain why you can use the IFT!)



2. Use Lagrange multipliers to find the point on the surface defined by  $x^2 + 2y^2 + 3z^2 = 1$  that maximizes the value of  $x^3 + y^3 + z^3$ .

3. Compute the volume of the solid bounded by the surfaces  $y=-1,\,y=x+1,\,y=1-x,$  z=0, and  $z=x^2+y^2.$ 

4. The region D in the xy-plane is bounded by the four lines with equations 2x + y = 1, 2x + y = 3, y - 2x = 2, y - 2x = 4. Find a convenient change of variables function (using new variables u and v), transform the integral below to an integral over a rectangle in the new coordinate plane, and then evaluate that integral.

$$\iint_D (16x^2 - 4y^2) \, dx \, dy$$

- 5. Set up the following, but <u>do not evaluate</u>.
  - (a) A single, iterated triple integral in cylindrical coordinates representing the z coordinate of the centroid of the mass inside the unit sphere, above the xy-plane, and below the plane z=1/2, with density  $\delta=x^2+z^4$ .

(b) A single, iterated double integral representing the moment of inertia around the x-axis of the lamina which is the part of the unit disk above the graph of y = |x|, with density given by  $\delta = e^{xy}$ . (Hint: Choose an appropriate coordinate system for your integral.)

(c) A single, iterated triple integral representing the mass of the region contained inside the sphere of radius a with center at (0,0,a), and below the cone z=r. (Hint: Choose an appropriate coordinate system for your integral.)