EXAM 3

Math 103, Fall 2005, Clark Bray.

You have 50 minutes.

No notes, no books.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

Good luck!

Name ____________________________________

ID number________________________

1. ____________ (/20 points)

2. ____________ (/20 points)

3. ____________ (/30 points)

4. ____________ (/15 points)

5. ____________ (/15 points)

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ________________________

Total ____________ (/100 points)

1. (a) Let S_1 and S_2 be the spheres of radius a with centers at $\begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ a \\ 0 \end{bmatrix}$, respectively. Derive the spherical equations of S_1 and S_2.

(b) Let D be the set of points interior to both S_1 and S_2. Suppose that the density inside D is given by the function $\delta(\vec{x})$. Write down, but do not evaluate, an iterated integral in spherical coordinates representing the mass contained in D.

2. (a) Show by direct computation that if C is the line beginning at $\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$ and ending at $\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$, then

$$\int_C x\,dy = \bar{x}\Delta y$$

(b) Let P be the hexagon with vertices (moving counterclockwise around P) at the points $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -4 \end{bmatrix}, \begin{bmatrix} 5 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Use the result above to compute the area of P, and make sure to explain how Green's Theorem relates that line integral to area.

3. (a) Let $\vec{F}$ be the field $\begin{bmatrix} e^{yz} \\ xze^{yz} \\ xye^{yz}+1 \end{bmatrix}$, and C the oriented curve defined parametrically by $\vec{r} = \begin{bmatrix} t^3 \\ t^2 \\ t \end{bmatrix}$, $t \in [0,1]$. Compute

$$\int_C \vec{F} \cdot d\vec{r}$$

(b) Let S be the closed surface defined by $\rho = 2 + \sin^3(\theta)\sin(6\phi)$, and let $\vec{F}$ be the field $\begin{bmatrix} y^2z^3 \\ 2yz \\ xy - z^2 \end{bmatrix}$. Compute

$$\iint_S \vec{F} \cdot d\vec{S}$$

4. Let S be the part of the graph $z = x^2 + y^2$ directly above the unit circle in the xy-plane, oriented upwards, and let $\vec{F}$ be the field $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$. Parametrize S, and then with that parametrization directly compute (without using any boundary theorems)

$$\iint_S \vec{F} \cdot d\vec{S}$$

5. Let $\vec{F}$ be the field $\begin{bmatrix} x^2z^3 \\ e^y(z^2+1) \\ xyz^2 \end{bmatrix}$, and C the oriented curve defined parametrically by $\vec{r} = \begin{bmatrix} 1+e^{\sin(t)} \\ \cos^3(t) \\ 1 \end{bmatrix}$, $t \in [0, 2\pi]$. Compute

$$\int_C \vec{F} \cdot d\vec{r}$$