

# EXAM 2

Math 103, Fall 2005, Clark Bray.

You have 50 minutes.

No notes, no books.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING  
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

Good luck!

Name \_\_\_\_\_

ID number \_\_\_\_\_

1. \_\_\_\_\_ (/20 points)

2. \_\_\_\_\_ (/20 points)

3. \_\_\_\_\_ (/20 points)

“I have adhered to the Duke Community  
Standard in completing this  
examination.”

4. \_\_\_\_\_ (/20 points)

Signature: \_\_\_\_\_

5. \_\_\_\_\_ (/20 points)

Total \_\_\_\_\_ (/100 points)

1. (a) Compute the value of  $D_{\vec{v}}f(\vec{a})$  in terms of  $p$ ,  $q$ , and  $r$ , where

$$f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} xyz \\ x^2y - z^2 \end{bmatrix} \quad \text{and} \quad \vec{a} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{and} \quad \vec{v} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

- (b) Suppose that we require  $p^2 + 4q^2 + r^2 \leq 4$ ; what velocity vector  $\vec{v}$  in the domain causes  $f_2$  to increase most quickly?

2. Consider the functions  $f : \mathbb{R}^2 \rightarrow \mathbb{R}^3$  and  $g : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ , where

$$f \left( \begin{bmatrix} s \\ t \end{bmatrix} \right) = \begin{bmatrix} s^3 t^2 - 2t \\ 2t^2 - s \\ t^4 - 2st^2 \end{bmatrix} \quad \text{and} \quad g \left( \begin{bmatrix} x \\ y \\ z \end{bmatrix} \right) = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix}$$

Suppose that when composed as  $g \circ f$ , we know that  $(\partial g_2 / \partial t)(1, 1) = 12$ ; what is  $(\partial g_2 / \partial y)(-1, 1, -1)$ ?

3. Evaluate the following, using any techniques from this course.

(a)  $\iint_R 2x^2y \, dA$ , where  $R$  is the part of the unit disk in the third quadrant.

(b)  $\iint_D \sin(x + y) \, dA$ , where  $D$  is the entire unit disk.

4. Write down, but do not evaluate, explicit iterated integrals representing the following quantities. Make sure that you clearly demonstrate how you arrived at your result.

(a) The centroid of the region in the second quadrant below the line  $y = 2x + 2$ .

(b) The mass of the region in  $\mathbb{R}^3$  bounded by the surfaces  $x^2 - y + z^2 = 0$  and  $x^2 + y + z^2 = 2$ , where the density is given by  $\delta = x^2 + y^2 + z^2$ .

5. Compute the following integral using the given change of variables.

$$\iint_D \left(2 - \frac{x^2}{y}\right) \left(8x + \frac{2x^3}{y^2}\right) dx dy$$

$D$  is the region in the first quadrant bounded by  $y = x^2$ ,  $3y = x^2$ ,  $x^2 + 2y^2 = 1$ ,  $x^2 + 2y^2 = 2$ ; use the variables  $u = x^2/y$ ,  $v = x^2 + 2y^2$ .