## EXAM 1

> Math 103, Fall 2005, Clark Bray.
> You have 50 minutes.
> No notes, no books, no calculators.
> YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT

Good luck!

Name $\qquad$
ID number $\qquad$

1. $\qquad$ (/30 points)
2. $\qquad$ (/20 points)
"I have adhered to the Duke Community Standard in completing this examination."
3. $\qquad$ (/30 points)

Signature: $\qquad$
4. $\qquad$ (/20 points)

Total $\qquad$ (/100 points)

1. (a) The parallelopiped $P$ has one vertex at the origin and the opposite vertex at $(1,3,0)$. Two of the edges touching the origin are defined by the vectors $(1,1,2)$ and $(1,1,-2)$, respectively. What is the volume of $P$ ?
(b) Find the equation for the plane $L$ in $\mathbb{R}^{3}$ that is parallel to and equidistant from the two lines

$$
\left[\begin{array}{l}
1 \\
2 \\
5
\end{array}\right]+t\left[\begin{array}{l}
1 \\
0 \\
1
\end{array}\right] \quad \text { and } \quad\left[\begin{array}{c}
-1 \\
4 \\
1
\end{array}\right]+t\left[\begin{array}{l}
0 \\
1 \\
0
\end{array}\right]
$$

2. The point $\vec{x}(t)$ moves along the path shown below according to the parametric equations

$$
x(t)=t^{3}-2 t+4 \quad y(t)=t^{3}-t+6
$$



Find the area between this curve, the positive part of the $x$ axis, and the line $x=8$.
3. Compute the following:
(a) $\frac{\partial}{\partial z}\left(\left(x^{2}+1\right)^{x-y} z^{x y}\right)$
(b) The slope of the tangent line to the cross-section of the graph of $f(x, y)=x^{2}+y^{2}$ by the vertical plane defined by the line $2 x+3 y=6$, at the point $(3,0)$.
(c) The sun causes a point $(x, y, z)$ to cast a shadow on the $x y$-plane at the point $(x-z, y-2 z, 0)$. Find the instantaneous velocity of the shadow of a point that is at $(1,2,4)$ and moving with velocity $(3,2,6)$.
4. The linear transformation $T$ has

$$
\begin{aligned}
& \qquad T\left(\vec{e}_{1}\right)=\left[\begin{array}{l}
2 \\
0 \\
3
\end{array}\right] \text { and } T\left(\vec{e}_{2}\right)=\left[\begin{array}{l}
0 \\
1 \\
0
\end{array}\right] \text { and } T\left(\vec{e}_{3}\right)=\left[\begin{array}{l}
0 \\
1 \\
3
\end{array}\right] \\
& \text { Compute }(T \circ T)\left(\left[\begin{array}{c}
1 \\
-1 \\
2
\end{array}\right]\right)
\end{aligned}
$$

