## EXAM 2

Math 103, Summer 2005 Term 2, Clark Bray.
You have 75 minutes.
No notes, no books, no calculators.
YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT

Good luck!

Name $\qquad$
ID number $\qquad$

1. $\qquad$ (/20 points)
2. $\qquad$ (/20 points)
3. $\qquad$ (/20 points)
4. $\qquad$ (/20 points)
"I have adhered to the Duke Community Standard in completing this examination."

Signature: $\qquad$
5. $\qquad$ (/20 points)

[^0]1. Consider the function $f(x, y)=x+y^{2}$, and the point $\vec{a}=(1,2)$. Of course the partial derivatives of $f$ at $\vec{a}$ are continuous, so $f$ is continuously differentiable at $\vec{a}$ and therefore also differentiable at $\vec{a}$. However in this problem, we will ignore that observation and prove directly from the definition that $f$ is differentiable at $\vec{a}$.
(a) Without assuming $f$ is differentiable, compute $D_{\vec{v}} f(\vec{a})$ in terms of the components of $\vec{v}=\left(v_{1}, v_{2}\right)$ and then show that there is a linear transformation $T$ (in other words, a matrix $A$ ) such that at this point $\vec{a}$,

$$
D_{\vec{v}} f(\vec{a})=T(\vec{v})=A \vec{v}
$$

(b) Show (by direct computation of the limit) that for the appropriately chosen linear transformation, the limit of the relative error does equal zero, and thus that $f$ is indeed differentiable.
2. Suppose we have $f, g: \mathbb{R}^{2} \rightarrow \mathbb{R}^{2}$, and suppose we know the following:

$$
f\left(\left[\begin{array}{l}
x \\
y
\end{array}\right]\right)=\left[\begin{array}{c}
5 e^{x y} \\
x-y
\end{array}\right] \quad \text { and } \quad g\left(\left[\begin{array}{l}
1 \\
2
\end{array}\right]\right)=\left[\begin{array}{l}
1 \\
0
\end{array}\right] \quad \text { and } \quad J_{g,\left[\begin{array}{l}
1 \\
2
\end{array}\right]}=\left(\begin{array}{ll}
2 & 7 \\
1 & 3
\end{array}\right)
$$

Compute $J_{f \circ g,}\left[\begin{array}{l}1 \\ 2\end{array}\right]$
3. We have $f:\left(D \subset \mathbb{R}^{3}\right) \rightarrow \mathbb{R}^{1}$ with $f(x, y, z)=(x+y+z)^{2}$ and $D=\left\{x^{2}+4 y^{2}+9 z^{2} \leq 1\right\}$. Find the point or points in the domain, if any exist, that attain the absolute maximum value of $f$ on the domain $D$.
4. Let $R$ be the region in the $x y$-plane bounded by the lines $y=x, y=x+1, y=0$, and $y=3$. Find the volume that is above $R$ and below the graph of the function $f(x, y)=y+x y+10$.
5. Find the centroid of the region in the first quadrant of the $x y$-plane, bounded by $y=x^{2}$, $x=0$, and $y=1$.


[^0]:    Total $\qquad$ (/100 points)

