EXAM 1

Math 103, Summer 2005 Term 2, Clark Bray.

You have 75 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT

Good luck!

Name ____________________________________

ID number________________________

1. ____________ (/20 points)

2. ____________ (/20 points)

3. ____________ (/20 points)

4. ____________ (/20 points)

5. ____________ (/20 points)

Total ____________ (/100 points)

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ________________________

1. (a) Find an equation of the unique plane that contains the point $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and is perpendicular to each of the planes below:

$$\begin{aligned} 3x + y + 2z &= 2 \\ x + y + 5z &= 1 \end{aligned}$$

(b) What is the function whose graph is the plane from part (a)?

2. Consider the parametric curve in the plane defined by

$$\vec{r}(t) = \begin{bmatrix} 1/(1+t^2) \\ t^3 - t \end{bmatrix}$$

(a) Find the velocity vector as a function of t.

(b) Between $t = -1$ and $t = 1$, this parametric curve traces out (clockwise) a closed loop that starts and ends at the origin, and does not intersect itself anywhere else. Write down (but do not evaluate!) a single integral representing the area of that loop, and make sure to explain your reasoning.

3. Consider the function f and vectors $\vec{p}$ and $\vec{q}$ given by

$$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} xy \\ x^2 + y^2 \end{bmatrix} \quad \text{and} \quad \vec{p} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad \vec{q} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

(a) Compute, directly from the definition, the value of $D_{\vec{q}}f(\vec{p})$.

(b) Compute the value of $D_{\vec{q}} f(\vec{p})$ using gradient vectors.

(c) At the point $\vec{p}$, in what unit vector direction is the second component of f increasing the fastest?

4. We have a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, and all we know about it is that

$$T\left(\begin{bmatrix}1\\1\end{bmatrix}\right) = \begin{bmatrix}3\\4\end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix}1\\-1\end{bmatrix}\right) = \begin{bmatrix}1\\0\end{bmatrix}$$

(a) Find the matrix that represents T.

(b) Find the matrix that represents $T \circ T$.

5. What is the value, if it exists, of the limit below?

$$\lim_{\vec{x}\to\vec{0}} \frac{x^3y - xy^3}{(x^2+y^2)^2}$$