

EXAM 2

Math 103, Summer 2005 Term 1, Clark Bray.

You have 75 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT

Good luck!

Name _____

ID number _____

1. _____ (/20 points)

2. _____ (/20 points)

3. _____ (/20 points)

4. _____ (/20 points)

5. _____ (/20 points)

“I have adhered to the Duke Community
Standard in completing this
examination.”

Signature: _____

Total _____ (/100 points)

1. Compute the general Jacobian matrix J_f for each of the functions below:

(a) $f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x^2y \\ y^3 \\ y^2 - e^x \end{bmatrix}$

(b) $f\left(\begin{bmatrix} u \\ v \\ w \end{bmatrix}\right) = \begin{bmatrix} u^2 - vw \\ uvw - e^{uvw} \end{bmatrix}$

2. Find the point (or points) in the unit disk in \mathbb{R}^2 that attains the maximum value on that domain of

$$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = x^2 - x + y^2 - y$$

3. What is the volume in \mathbb{R}^3 above the xy -plane, bounded by the planes $x = 0$, $y = x$, $x + y = 2$, and $z = 2x + 3y + 1$?

4. Suppose that for a given function $f : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ and a given point $\vec{a} \in \mathbb{R}^2$, we know that for any vector $\vec{v} = (v_1, v_2, v_3)$,

$$D_{\vec{v}}f(\vec{a}) = \begin{bmatrix} v_1 + 2v_3 \\ v_2^3/\|\vec{v}\|^2 \\ 7v_2 \end{bmatrix}$$

Explain, with specific examples to support your reasoning, how you know that this function cannot be differentiable at this point.

5. Suppose we have differentiable functions $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ and $g : \mathbb{R}^2 \rightarrow \mathbb{R}^3$, with

$$f\left(\begin{bmatrix} s \\ t \end{bmatrix}\right) = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad \text{and} \quad g\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

and suppose we know the following value of f , and three vector derivatives:

$$\begin{aligned} f(\vec{a}) &= \vec{b} & D_{\begin{bmatrix} 1 \\ 1 \end{bmatrix}} g(\vec{b}) &= \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix} \\ D_{\begin{bmatrix} 2 \\ 5 \end{bmatrix}} f(\vec{a}) &= \begin{bmatrix} 2 \\ 0 \end{bmatrix} & D_{\begin{bmatrix} 1 \\ -1 \end{bmatrix}} g(\vec{b}) &= \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} \end{aligned}$$

Use derivative transformations and the chain rule to compute

$$2 \frac{\partial w}{\partial s}(\vec{a}) + 5 \frac{\partial w}{\partial t}(\vec{a})$$

(Hint: Rephrase the given vector derivatives in terms of derivative transformations. Then compute $D_{(g \circ f), \vec{a}}\left(\begin{bmatrix} 2 \\ 5 \end{bmatrix}\right)$ by first using the chain rule, and then the linearity of $D_{g, \vec{b}}$; then relate this to the quantity you are supposed to compute.)