EXAM 1

Math 103, Summer 2005 Term 1, Clark Bray.

You have 75 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT

Good luck!

Name ________________________________

ID number______________________

1. __________ (/15 points)

2. __________ (/15 points)

3. __________ (/20 points)

4. __________ (/20 points)

5. __________ (/15 points)

6. __________ (/15 points)

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ______________________

Total __________ (/100 points)

1. The plane P is parallel to the vectors $\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$, and the plane Q is parallel to the vectors $\begin{bmatrix} 2 \\ 2 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$. Both planes contain the point $\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$.

   Find a parametric representation of the line that is the intersection of P and Q.

2. The line L goes through the points $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ 1 \end{bmatrix}$. Find functions f, g, and h such that: L is the graph of f, L is a level set of g, and h is a parametric representation of L.

3. Compute the following by any methods from this course:

(a)

$$\frac{d}{dt}\begin{bmatrix} t^3 \\ t^2 + e^t \\ e^{t^2} \end{bmatrix}$$

(b)

$$\frac{\partial}{\partial y} \ln(x^2 y) e^{z^2 y e^x}$$

(c)

$$D_{\vec{v}}\left(y e^{x^2+yz}\right)(\vec{a}) \text{ where } \vec{v} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} \text{ and } \vec{a} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

4. Write as a single variable derivative of a function of t, and then compute directly from that form, the directional derivative of the function $f(x, y, z) = x + 2y + 3z$ at the point $\vec{a} = \begin{bmatrix} 4 \\ 3 \\ 7 \end{bmatrix}$, in the direction that points from $\vec{a}$ toward the point $\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$.

5. The temperature (in degrees Celsius) in a room is given by the function

$$T(\vec{x}) = (x^2 + y^2)e^z$$

If a fly is at the point $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, in what (unit vector) direction should the fly travel to increase his temperature the most quickly? And, what is the rate of increase of the temperature in that direction?

6. The linear transformation T rotates vectors in the right hand direction by $\pi/2$ around the vector $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$; and the linear transformation S rotates vectors in the right hand direction by $\pi/2$ around the vector $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$. What is the matrix representing the linear transformation $S \circ T$?