EXAM 4

Math 103, Fall 2004, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT

Good luck!

Name ________________________________

ID number______________________

1. __________ (/40 points)

2. __________ (/30 points)

3. __________ (/15 points)

4. __________ (/15 points)

Total __________ (/100 points)

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ______________________

1. In each of the following cases, use any techniques from this course to compute

$$\int_C \overrightarrow{F} \cdot d\overrightarrow{r}$$

   (a) $\overrightarrow{F} = (y, x)$, $C =$ the arc of the unit circle going counter-clockwise from $(1, 0)$ to $(0, 1)$.

   (b) $\overrightarrow{F} = (x + y, y)$, $C =$ the entire unit circle, oriented counter-clockwise.

(c) $\overrightarrow{F} = (x+y, y)$, $C =$ the arc of the unit circle going counter-clockwise from $(1,0)$ to $(0,1)$.

(d) $\overrightarrow{F} = (2x, z, y)$, $\overrightarrow{r} = (t(t-1)e^{t^2}\cos t, t + t^2(t-1), (t-1)\sin t)$, $t \in [0,1]$

2. In each of the following cases, use any techniques from this course to compute

$$\iint_S \overrightarrow{F} \cdot \overrightarrow{n}\, dS$$

(a) $\overrightarrow{F} = \nabla \times (x, 0, x^2yz^3)$, $S = \{x^2 + 4y^2 + 5z^2 = 21, z \geq -1\}$, oriented such that the normal vector is $(0, 0, 1)$ at the point where z is greatest.

(b) $\overrightarrow{F} = (x + xy, y + yz, z + zx)$, S is the unit sphere.

(c) $\overrightarrow{F} = (0, y+1, 0)$, S is the disk of radius 1 in the plane $y + z = 0$ with center at (0,0,0), and upward normal.

3. Let S be the part of the sphere $x^2 + y^2 + (z-4)^2 = 25$ that is above the xy-plane, oriented with the outward pointing normal from the sphere itself. Let $\overrightarrow{F} = (ze^{y^2}, xe^z, y)$. Determine the value of

$$\iint_S \overrightarrow{F} \cdot \overrightarrow{n}\, dS$$

4. Suppose that $\overrightarrow{F}(x, y, z) = (P(x, y), Q(x, y), 0)$, and that C is the closed curve parametrized by $\overrightarrow{r}(t) = (\cos^3(t), \cos^3(t), \sin^3(t))$, $t \in [0, 2\pi]$. Show that

$$\oint_C \overrightarrow{F} \cdot d\overrightarrow{r} = 0$$