EXAM 2

Math 103, Fall 2004, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT

Good luck!

Name ________________________________

ID number______________________

1. ___________ (/20 points)

2. ___________ (/40 points)

3. ___________ (/20 points)

4. ___________ (/20 points)

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ______________________

Total ___________ (/100 points)

1. Determine the values of the following limits, if they exist, making sure to justify your answers:

   (a)
$$\lim_{(x,y)\to(0,0)} \frac{x^2 + xy}{x^2 + y^2}$$

   (b)
$$\lim_{(x,y)\to(0,0)} \frac{2x^2 y}{x^4 + y^2}$$

2. Let $f(x, y, z) = x^2y + y^2z^3$, and $\overrightarrow{p}(t) = (t^2, t^3 + 1, t^6 - 2)$

   (a) Compute ∇f.

   (b) Compute $D_{\overrightarrow{u}} f(\overrightarrow{a})$ with $\overrightarrow{u} = (2, 3, 6)/7$ and $\overrightarrow{a} = (1, 2, -1)$.

(c) Compute $\overrightarrow{p}(1)$ and $\overrightarrow{p}'(1)$.

(d) Use the chain rule to compute $\frac{d}{dt} f(\overrightarrow{p}(t))|_{t=1}$ (without referring to a directional derivative); then explain what your answer has to do with the directional derivative computed in part (b).

3. (a) Find a normal vector to the graph of the function

$$z = f(x, y) = x^4y^2 - y^3 + x$$

at the origin. *(Hint: This graph can be thought of as a level set of some function* $g : \mathbb{R}^3 \to \mathbb{R}^1$*.)*

(b) What unit vector $\overrightarrow{u}$ maximizes the directional derivative of f at the origin?

4. Let D be the collection of points in $\mathbb{R}^3$ satisfying

$$4x^2 + 2y^2 + 5z^2 - 6xz + 4yz \leq \frac{3}{4}$$

What is the "highest" (greatest value of z) point in this set?