EXAM I

Math 103, Fall 2004, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT

Good luck!

Name ______________________________

ID number ____________________

1. __________ (/40 points)

2. __________ (/30 points)

3. __________ (/30 points)

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ____________________

Total __________ (/100 points)

1. Consider the following three vectors:

$$\overrightarrow{u} =< 3, 4, 12 > \qquad \overrightarrow{v} =< 2, 3, 6 > \qquad \overrightarrow{w} =< 3, 6, 6 >$$

   (a) Find the magnitude of each of these vectors.

   (b) Find the angle between $\overrightarrow{u}$ and $\overrightarrow{v}$.

(c) Find the volume of the unique parallelepiped that is defined by $\overrightarrow{u}$, $\overrightarrow{v}$, and $\overrightarrow{w}$.

(d) Find the equation of the unique plane that passes through the points $\overrightarrow{u}$, $\overrightarrow{v}$, and $\overrightarrow{w}$.

2. Consider the curve parametrized by

$$< t, t^3 - t > \qquad t \in [-2, 2]$$

(a) Show the process of rewriting the expression

$$s = \int ds$$

to arrive at an integral, in terms of t, that represents the length of this curve.

(b) Write down a single integral, in terms of t, that represents the area between this curve and the lines

$$x = 4 \quad y = -6 \quad y = 6$$

Make sure to explain how you know that your integral represents the desired area. (Note: In order to get a single integral, your "slices" of area will have to be horizontal!)

3. The Enterprise is damaged, and its position sensors are not working; however, the computer still has complete control over the amount of force the engines put out.

   (a) Spock proposes firing the engines in order to realize a (time-dependent) acceleration vector of
   $$a(t) =< 6t, 6t - 24t^2, 4 >$$
   over the time interval $[0, 10]$.

   Assuming that the ship's initial position and velocity are the zero vectors, find an expression for the ship's position as a function of time.

   (b) Following this path over the given time interval, will the ship cross the boundary of the Neutral Zone, with equation defined by
   $$-10x - y + 10z = 5000$$
   (Hint: What are the starting and ending values for $-10x - y + 10z$ as the ship moves from $t = 0$ to $t = 10$?)

(c) At the time $t = 1$, what are the normal and tangential components of the acceleration? What is the curvature of the path at that point?