EXAM 2
Math 212, 2018 Summer Term 1, Clark Bray.

Name:_____________________________  Section:____  Student ID:__________

GENERAL RULES

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

No notes, no books, no calculators. Scratch paper is allowed, but (1) it must be from the instructor, (2) it must be returned with the exam, and (3) it will NOT be graded.

All answers must be reasonably simplified.

All of the policies and guidelines on the class webpages are in effect on this exam.

WRITING RULES

Do not write anything on the QR codes or near the staple.

Use dark pen only – no pencils.

Work for a given question can ONLY be done on the front or back of the page the question is written on.

DUKE COMMUNITY STANDARD STATEMENT

“I have adhered to the Duke Community Standard in completing this examination.”

Signature: ___________________________
(Nothing on this page will be graded!)
1. (20 pts) The differentiable function $f : \mathbb{R}^3 \rightarrow \mathbb{R}$ representing temperature as a function of location has gradient at the point $(7, 8, 9)$ given by $\nabla f \bigg|_{(7,8,9)} = (3, 6, 2)$.

Bob is in an airplane and at a given moment he is at the location $(7, 8, 9)$ and moving with velocity $(-1, 2, 2)$.

(a) What is the rate of change of temperature with respect to time at this moment for Bob?

(b) What is the rate of change of temperature with respect to distance travelled at this moment for Bob?

(c) What unit vector indicates the direction he should have been going at that point if he had wanted to maximize the rate of change of temperature?

(d) Had he been going in the direction referred to in part (c), with speed equal to 14, what would have been the rate of change of temperature with respect to time?
(extra space for question from other side)
2. (20 pts) The functions $f, g, h : \mathbb{R}^2 \to \mathbb{R}^2$ are defined by

$$f(x, y) = \begin{pmatrix} x^2 - 2x + y \\ 3x - 4y \end{pmatrix} \quad g(x, y) = \begin{pmatrix} 2x + 3y \\ 4x - 5y + 3y^2 \end{pmatrix} \quad h(x, y) = \begin{pmatrix} x - y \\ x + y \end{pmatrix}$$

(a) Let $m = g \circ f$ with $(p, q) = m(x, y)$. Use the chain rule to find an expression for $\frac{\partial q}{\partial x}$.

(Do NOT explicitly compose the functions.)

(b) Let $n = h \circ g \circ f$ with $(r, s) = n(x, y)$. Use the chain rule to find an expression for $\frac{\partial s}{\partial y}$.

(Do NOT explicitly compose the functions.)
(extra space for question from other side)
3. (20 pts) The region $R \in \mathbb{R}^3$ is defined by the inequalities $x - y + z \leq 1$, $x - y - z \leq 1$, $x - y \geq 0$, $x + y \leq 3 + z$, $x + y \geq -2 - x + y$. Set up a triple iterated integral that computes the volume of $R$. (Hint: Start with a change of variables $u = x + y$, $v = x - y$, $w = z$ on the integral $\iiint 1 \, dx \, dy \, dz$.) You do not have to compute the iterated integral.
(extra space for question from other side)
4. (20 pts) Compute the mass of the solid bounded by \( z = x^2 + y^2 \), \( x^2 + y^2 = 1 \), and \( z = 0 \), given that mass is distributed with density \( \delta = z \).
(extra space for question from other side)
5. (20 pts) The surface $S$ is defined by $x^2 - y^2 + z = 1$, $|x| \leq 1$, $|y| \leq 1$. Set up an iterated integral that computes $\iint_S 3z \, dS$. You do not have to compute the iterated integral.
(extra space for question from other side)