EXAM 1
Math 212, 2017 Summer Term 1, Clark Bray.
You have 75 minutes.
No notes, no books, no calculators.
YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING. All answers must be simplified. All of the policies and guidelines on the class webpages are in effect on this exam.
Good luck!
Name ________________________________

“I have adhered to the Duke Community Standard in completing this examination.”

1. _________

2. _________ Signature: _______________________

3. _________

4. _________

5. _________

6. _________

Total Score ___________ (/100 points)
1. (20 pts) In this question we consider the vectors

\[ \vec{u} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \quad \vec{v} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \quad \vec{w} = \begin{pmatrix} 5 \\ 1 \\ 1 \end{pmatrix} \]

(a) Compute \( \vec{v} \times \vec{w} \).

(b) Compute the area of the parallelogram defined by \( \vec{v} \) and \( \vec{w} \).

(c) Compute the determinant of the matrix whose rows are \( \vec{u}, \vec{v}, \vec{w} \), in that order.

(d) Find the volume of the parallelepiped defined by \( \vec{u}, \vec{v}, \vec{w} \), and decide if that listing is in right hand or left hand order.

(e) Using any of the results above, find the value of \( \vec{u} \cdot (\vec{v} \times \vec{w}) \) with as little computational effort as possible. Explain your reasoning.
2. (10 pts) The point $P$ has rectangular coordinates $(-1, \sqrt{3}, 2)$. There are unique spherical coordinates $\rho, \phi, \theta$ for $P$ that satisfy the requirements $0 < \phi < \pi$ and $\pi < \theta < 2\pi$. Find these values of $\rho, \phi, \theta$.

3. (10 pts) Find the polar equation for the circle of radius 2 with center at $(0, -2)$. 
4. (20 pts) We define the function \( \text{Tan} : (-\pi/2, \pi/2) \to \mathbb{R} \) by \( \text{Tan} x = \tan x \). Note that \( \text{Tan} \) is defined only for \( -\pi/2 < x < \pi/2 \).

The curve \( C \) has equation \( y = \text{Tan} x \). We would like to find the equation of the surface \( S \) obtained by rotating this curve around the \( y \)-axis.

(a) Draw a curve \( D \) in the \( x \geq 0 \) portion of the \( xy \)-plane whose rotation around the \( y \)-axis is the same surface \( S \).

(b) Find an equation for the curve \( D \) in the given half plane. (Hint: Think about how squaring might help you with the signs.)

(c) Find an equation for the surface \( S \).
5. (20 pts) The curve \( C \) in the \( xy \)-plane has equation \( y = e^x - yx^2 \).

(a) \( C \) is the graph of a function \( h : \mathbb{R}^a \rightarrow \mathbb{R}^b \). Find \( a, b \), and a formula for computing \( h \).

(b) \( C \) is a level set of a function \( m : \mathbb{R}^c \rightarrow \mathbb{R}^d \). Find \( c, d \), and a formula for computing \( m \).

(c) \( C \) is the image of a function \( r : \mathbb{R}^e \rightarrow \mathbb{R}^f \). Find \( e, f \), and a formula for computing \( r \).
6. (20 pts) The function \( f : \mathbb{R}^3 \rightarrow \mathbb{R}^3 \) is the composition of the following: (1) a reflection through the plane \( y = z \); followed by (2) a rotation around the \( y \)-axis by an angle of \( \pi/2 \), clockwise as seen from the positive part of the \( y \)-axis. Find an explicit formula for \( f(x, y, z) \).