EXAM 1
Math 212, 2015 Summer Term 1, Clark Bray.
You have 75 minutes.
No notes, no books, no calculators.
YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING. All answers must be simplified. All of the policies and guidelines on the class webpages are in effect on this exam.

Good luck!

Name ________________________________

1. ________  “I have adhered to the Duke Community Standard in completing this examination.”

2. ________

3. ________  Signature: __________________________

4. ________

5. ________

6. ________

7. ________

8. ________  Total Score __________ (100 points)
1. (15 pts) Find the area of the parallelogram defined by the vectors \((3, 2, 5)\) and \((1, -2, -1)\).

2. (15 pts) Find the equation of the plane that perpendicularly bisects the line segment between \((4, 1, 7)\) and \((5, 4, 2)\).
3. (10 pts) The curve $C$ in the $xy$-plane is described by the equation $y = x^4e^{-x^2}$. Find the equation of the surface $S$ obtained by rotating $C$ around the $y$-axis.

4. (10 pts) Find a function $h$ whose graph is a level set of the function $p : \mathbb{R}^3 \to \mathbb{R}$ defined by $p(x, y, z) = x^2y - xy^2 + z^3$. 
5. (10 pts) Find a parametrization of the surface with equation \( x(y^2 + 3) - y^2 e^y = yz \cos^2 z \).

6. (10 pts) Compute the limit below.

\[
\lim_{\vec{x} \to \vec{0}} \frac{x^3 y - xy^2}{x^2 + y^2}
\]
7. (15 pts) The linear transformations $S$ and $T$ are given below.

\[
S(x, y, z) = \begin{pmatrix} 3x - 2y + z \\ 2x - 2y \\ y + 3z \end{pmatrix} \quad \text{and} \quad T(x, y, z) = \begin{pmatrix} x - y - z \\ 3x + 2y + z \\ x - z \end{pmatrix}
\]

Find the matrix representing the linear transformation $S \circ T$, without explicitly composing these transformations.
8. (15 pts) The velocity of a particle in the plane is $\vec{v}(t) = (4e^{2t}, 3 \sin t)$, and its initial position is $(5, 6)$. Find the position of the particle as a function of time.