EXAM 2
Math 212, 2013 Summer Term 2, Clark Bray.
You have 75 minutes.
No notes, no books, no calculators.
YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.
Good luck!

Name ________________________________

“I have adhered to the Duke Community
Standard in completing this
examination.”

1. _________

2. _________ Signature: __________________

3. _________

4. _________

5. _________

6. _________

Total Score ___________ (/100 points)
1. (21 pts) In this problem we consider the function $g : \mathbb{R}^2 \to \mathbb{R}^1$ defined by

$$g(x, y) = xy e^x + xy^2$$

(a) Find the direction of fastest increase of this function at the point $(0, 2)$, and the slope of the tangent line to the cross section of the graph of $g$ in this direction.

(b) Find the directional derivative of this function at the point $(0, 2)$ with velocity $\vec{v} = (3, 2)$.

(c) Suppose that $x$ and $y$ are the usual functions of $r$ and $\theta$. Compute $\frac{\partial g}{\partial \theta}$ at the point $(0, 2)$ in the $xy$-plane.
2. (15 pts) In this problem we consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x, y) = \begin{cases} \frac{y^5 + 5x^4y - 10x^2y^3}{(x^2 + y^2)^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

(a) Compute the directional derivative of $f$ at the origin, with velocity $\vec{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

(b) What is the slope of the graph of this function in the direction defined by $\vec{v}$?
3. (24 pts) In this problem we will consider the integral \( \iint_D x^2 \, dx \, dy \), where \( D \) is the first quadrant part of the unit disk in the \( xy \)-plane.

(a) Write down, but do not evaluate, a double nested integral representing the above double integral.

(b) Explain how you know that \( \iint_D (x^2 - y^2) \, dx \, dy = 0 \) (which allows us to conclude that \( \iint_D x^2 \, dx \, dy = \iint_D y^2 \, dx \, dy \)).

(c) Compute \( \iint_D x^2 + y^2 \, dx \, dy \) using polar coordinates.

(d) Use (b) and (c) above to compute \( \iint_D x^2 \, dx \, dy \).
4. (14 pts) Write down, but do not evaluate, a triple nested integral that represents the population of bacteria in the solid bounded inside of both the sphere with equation \((x-1)^2 + (y+3)^2 + (z+1)^2 = 25\) and the cylinder with equation \(x^2 + z^2 = 4\), where the bacteria population density is known to be \(\rho(x, y, z) = e^{-(x^2+z^2)}\).
5. (14 pts) Write down, but do not evaluate, one double nested integral that represents the mass of the lamina that is the image in the $xy$-plane of the unit square in the $uv$-plane by the function $(x, y) = g(u, v) = (e^u \cos v, e^u \sin v)$, where the density of the lamina is known to be $\delta(x, y) = x^2 + y^2$.

6. (12 pts) Write down, but do not evaluate, a triple nested integral in spherical coordinates representing $\iiint_T x^2 \, dV$, where $T$ is the solid inside of the unit sphere and above the $xy$-plane.