Math 212, 2013 Summer Term 2, Clark Bray.

You have 75 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING. All answers must be simplified. All of the policies and guidelines on the class webpages are in effect on this exam.

Good luck!

Name ____________________________________________

1. __________

“I have adhered to the Duke Community Standard in completing this examination.”

2. __________

Signature: ____________________________

3. __________

4. __________

5. __________

6. __________

7. __________

Total Score ____________ (/100 points)

“I have adhered to the Duke Community Standard in completing this examination.”
1. (10 pts) Suppose that \( \vec{v} \) and \( \vec{w} \) are nonzero vectors that are not parallel. Use the determinant to show that \( \vec{v} \times \vec{w}, \vec{v}, \vec{w} \) is in right-hand order.

   Explain why this argument relies on \( \vec{v} \) and \( \vec{w} \) being nonzero vectors that are not parallel.

2. (15 pts) Find the symmetric equations for the line that is the intersection of the planes

   \[ 3x - 2y + 6z = 7 \quad \text{and} \quad 2x + y + z = 4. \]

   (Hint: \((1, 1, 1)\) might be useful in some way.)
3. (15 pts) Find the equation of the surface $S$ that is the unique (right circular) hyperboloid of two sheets such that:

(a) the vertices of $S$ are at the points $(0,3,0)$ and $(0,5,0)$
(b) the cross section of $S$ in the plane $y = 1$ is a circle of radius 1
4. (15 pts) The surface $S$ has equation $x^2y - ze^{2xy} = 3y^2$.

(a) Find a function $f$ whose graph is $S$.

(b) Find a function $g$ such that $S$ is a level set of $g$.

(c) Find a function $h$ that parametrizes $S$. 
5. (15 pts) Compute the following limit.

\[
\lim_{(x,y) \to (0,0)} \left( \frac{x^2 + y^2}{\sec(xy)} \right)
\]

6. (15 pts) Suppose that \( T : \mathbb{R}^3 \to \mathbb{R}^7 \) is a linear transformation represented by a matrix \( A \), and you are given

\[ T(3, 2, 4) = \vec{v} \quad \text{and} \quad T(0, 1, 2) = \vec{w} \]

Find the first column of \( A \).
7. (15 pts) Suppose that at the moment when \( t = 3 \), the parametric curve \( \vec{x} \) has position \((3, 2, 5)\) and velocity \((1, 2, 0)\), and the parametric curve \( \vec{y} \) has position \((1, 3, 1)\) and velocity \((0, 1, 1)\). Compute the rate of change of \( f(t) = \vec{x}(t) \cdot \vec{y}(t) \), and the velocity of the parametric curve \( \vec{z}(t) = \vec{x}(t) \times \vec{y}(t) \).