EXAM 2
Math 103, 2010-2011 Summer Term 1, Clark Bray.
You have 75 minutes.
No notes, no books, no calculators.
YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING. All answers must be simplified. All of the policies and guidelines on the class webpages are in effect on this exam.
Good luck!

Name ________________________________

ID number _______________________

1. __________

2. __________  “I have adhered to the Duke Community Standard in completing this examination.”

3. __________

4. __________  Signature: _______________________

5. __________

6. __________

7. __________

8. __________

9. __________  Total Score ___________ (/100 points)

10. __________
1. (10 pts) Consider the function \( f(x, y, z) = (xy^2z, \frac{x^2z}{y}) \). At the point \( \vec{a} = (1, 2, 3) \), compute the directional derivatives with each of the velocities below. (You may do this by any method discussed in this course.)

\[
\vec{v}_1 = \begin{pmatrix} 2 \\ 4 \\ 3 \end{pmatrix} \quad \text{and} \quad \vec{v}_2 = \begin{pmatrix} 1 \\ 6 \\ -3 \end{pmatrix} \quad \text{and} \quad \vec{v}_3 = \begin{pmatrix} -3 \\ -2 \\ 2 \end{pmatrix}
\]

2. (10 pts) A hill is in the shape of the graph of the function \( f(x, y) = e^{-(3x^2 + 7y^2)} \). At the point above (4, 1), what direction (in the \( xy \)-plane) points directly uphill? What is the slope of the hill at that point?
3. (10 pts) We consider here \( z = f(x, y) \) (where \( f \) is continuously twice differentiable), with \( x = r \cos \theta \) and \( y = r \sin \theta \). Compute \( \frac{\partial^2 z}{\partial \theta^2} \) in terms of \( r, \theta \), and the derivatives of \( z \) with respect to \( x \) and \( y \).

4. (10 pts) We consider here the surface described by the equation \( x^2y - xe^{yz} = -1 \). At the point \((1, 1, 0)\), decide if \( y \) can locally be viewed as a function of \( x \) and \( z \); if it can, compute \( \frac{\partial y}{\partial z} \).
5. (10 pts) Compute the double integral \( \int\int_{R} xy \, dA \), where \( R \) is the region in the \( xy \)-plane described by \( x^2 + y^2 \leq 4 \), \( x \geq 0 \), and \( y \geq -1 \).

6. (10 pts) Set up, but do not evaluate, a single triple nested integral representing \( \int\int\int_{R} f(x, y, z) \, dV \), where \( R \) is the region described by \( x^2 + y^2 + z^2 \leq 9 \) and \( y^2 + z^2 \leq 4 \).
7. (10 pts) Compute the integral $\iint_R x \, dA$, where $R$ is the square with vertices at (0, 0), (5, 12), (−12, 5), (−7, 17).
8. (15 pts) Set up, but do not evaluate, a triple nested integral in spherical coordinates representing \[ \iiint_R f(x, y, z) \, dV, \] where \( R \) is the region inside of the sphere of radius 1 centered at \((0, 0, 1)\) on the \( z \)-axis, and above the plane \( z = 1 \).
9. (15 pts) The curve $C$ is the counterclockwise oriented circle of radius 5 centered at the origin. Compute the line integral \( \int_C x^3 - y^3 \, ds \) by the most efficient means you can.