EXAM 1
Math 103, 2010-2011 Summer Term 1, Clark Bray.

You have 75 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING. All answers must be simplified. All of the policies and guidelines on the class webpages are in effect on this exam.

Good luck!

Name ____________________________________

ID number ______________________________

1. ________

2. ________

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7. ________

8. ________

9. ________

Total Score ___________ (/100 points)

10. ________

“I have adhered to the Duke Community Standard in completing this examination.”

Signature: ____________________________
1. (10 pts) Find the symmetric equations for the line \( L \) that passes through the point \( \vec{x}_0 = (1, 3, 4) \) and is parallel to the vector \( \vec{v} = (3, 1, 2) \).

2. (10 pts) The points \( \vec{x} \) on the line \( L \) from the previous problem are defined by the property that \( (\vec{x} - \vec{x}_0) \parallel \vec{v} \). What does this mean about the length of the cross product \( (\vec{x} - \vec{x}_0) \times \vec{v} \)? Use this observation to find a single quadratic equation for the line \( L \).
3. (10 pts) The surface $S$ has equation $x^2 + y^2 - 4z^2 = -16$. Draw a rough sketch of $S$. 
For the problems on this page, we consider the surface $B$ that is the bottom sheet ($z < 0$) of the surface $S$ on the previous page.

4. (10 pts) $B$ is the graph of a function $f$. Find this function $f$, and make sure to indicate the domain and target.

5. (10 pts) The surface $B$ is a level set of a function $g$. Find such a function $g$, and make sure to indicate the domain and target. Indicate also which level set of $g$ is this surface $B$.

6. (10 pts) The surface $B$ is parametrized by a function $h$. Find such a function $h$, and make sure to indicate the domain and target.
7. (10 pts) Compute the limit below, if it exists.

\[ \lim_{\vec{x} \to \vec{0}} \frac{xy - 3y^4}{x^2 + y^2} \]
8. (10 pts) Suppose we know the following about the linear transformations $S$ and $T$:

\[
S \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \quad S \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}, \quad T \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}, \quad T \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \quad T \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}
\]

Find the matrix that represents the composition $S \circ T$. 

9. (10 pts) Write the definition of the directional derivative $D_{\vec{v}}f(\vec{a})$.

10. (10 pts) Use the definition above to compute the directional derivative of the function $f(x, y, z) = xy - e^{xz}$ at the point $(0, 3, 2)$ with velocity $(2, 0, 3)$.