On Dark Matter, Spiral Galaxies, and the Axioms of General Relativity

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Colloquium in the Department of Mathematics

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Tuesday, May 10, 2016

The Puzzle of the Spirals

"Much as the discovery of these strange forms may be calculated to excite our curiosity, and to awaken an intense desire to learn something of the laws which give order to these wonderful systems, as yet, I think, we have no fair ground even for plausible conjecture."

Lord Rosse (1850)

"A beginning has been made by Jeans and other mathematicians on the dynamical problems involved in the structure of the spirals."

Curtis (1919)

"Incidentally, if you are looking for a good problem..."

Feynman (1963)

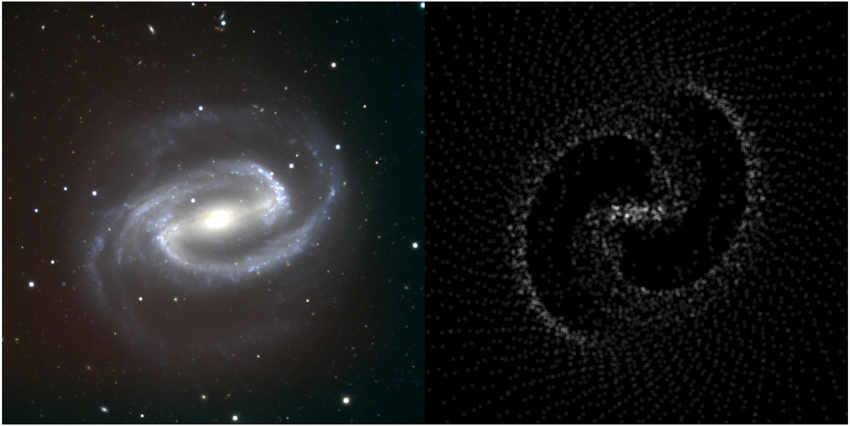
The Puzzle of the Spirals

"The old puzzle of the spiral arms of galaxies continues to taunt theorists. The more they manage to unravel it, the more obstinate seems the remaining dynamics. Right now, this sense of frustration seems greatest in just that part of the subject which advanced most impressively during the past decade - the idea of Lindblad and Lin that the grand bisymmetric spiral patterns, as in M51 and M81, are basically compression waves felt most intensely by the gas in the disks of those galaxies. Recent observations leave little doubt that such spiral "density waves" exist and indeed are fairly common, but no one still seems to know why.

To confound matters, not even the N-body experiments conducted on several large computers since the late 1960s have yet yielded any decently long-lived regular spirals."

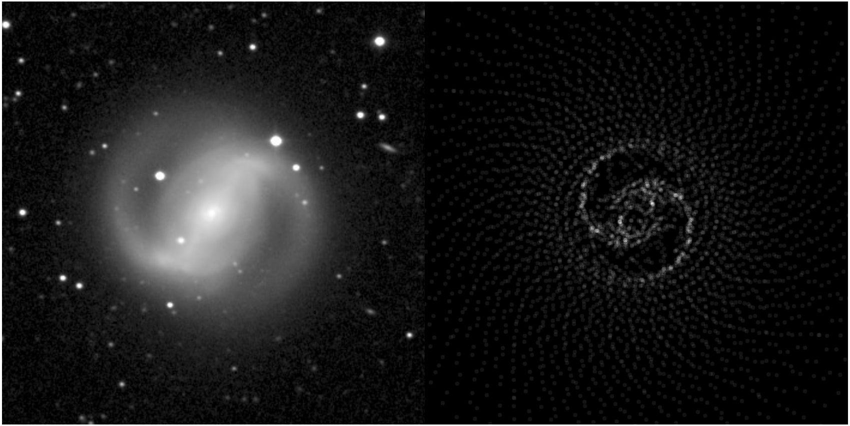
Toomre (1977)

Spiral Galaxy Simulation #1



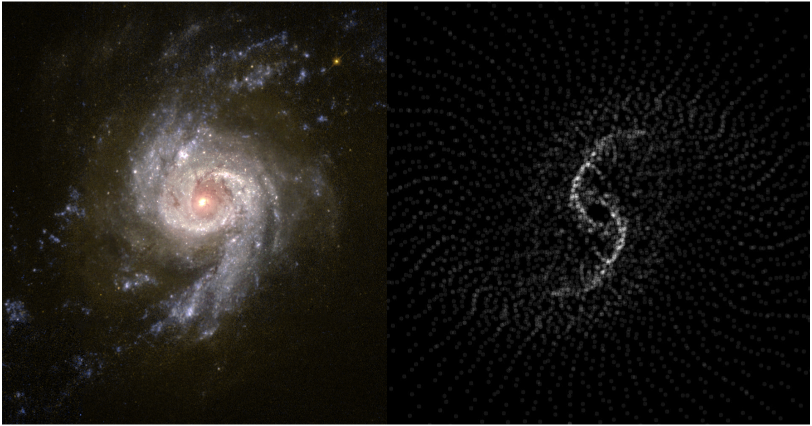
NGC1300 on the left, simulation on the right.

Spiral Galaxy Simulation #2



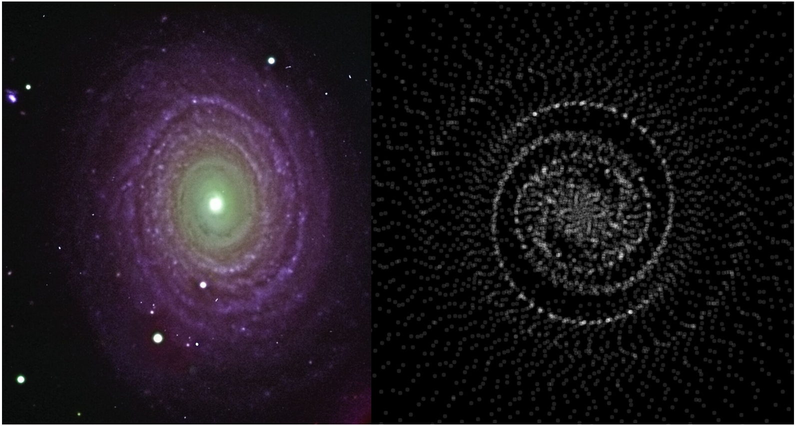
NGC4314 on the left, simulation on the right.

Spiral Galaxy Simulation #3



NGC3310 on the left, simulation on the right.

Spiral Galaxy Simulation #4



NGC488 on the left, simulation on the right.

Big Questions

There is roughly five times more dark matter in the universe than regular baryonic matter represented by the periodic table.

Also, most of the mass of galaxies is dark matter.

1. What is the nature of dark matter?

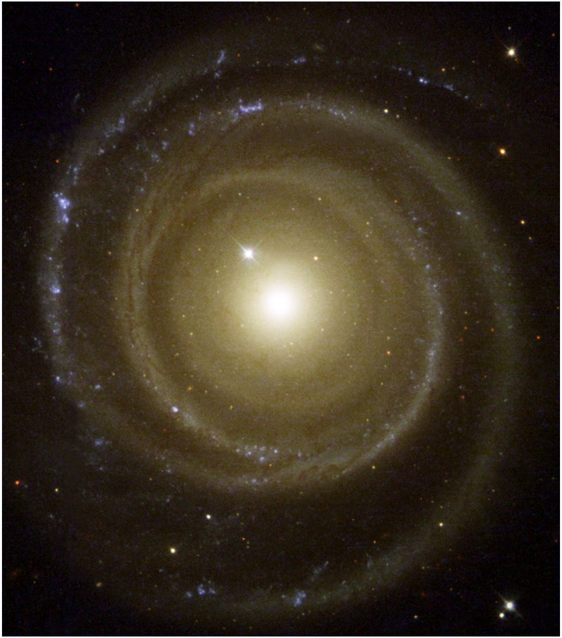
2. Does dark matter have something to do with spiral structure in galaxies?

Spiral Galaxy M81

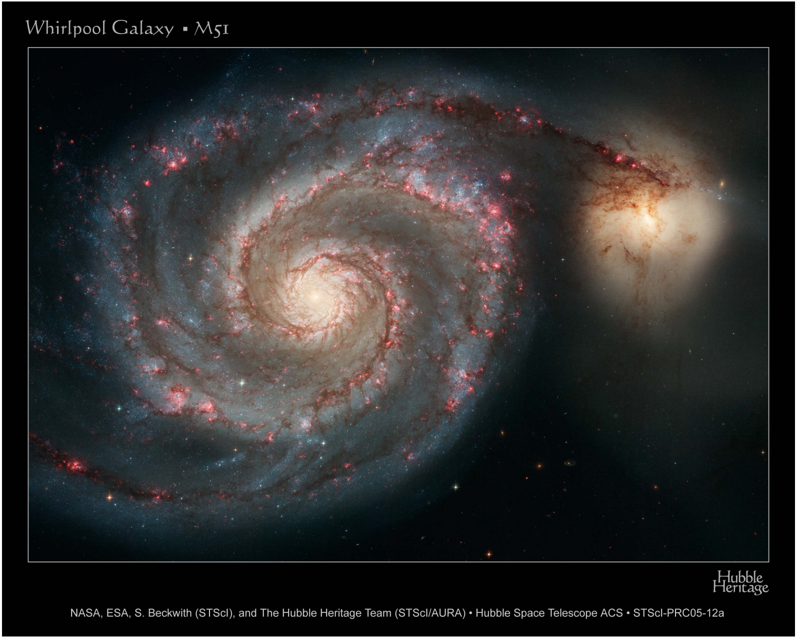
Spiral Galaxy M74

Spiral Galaxy NGC1365

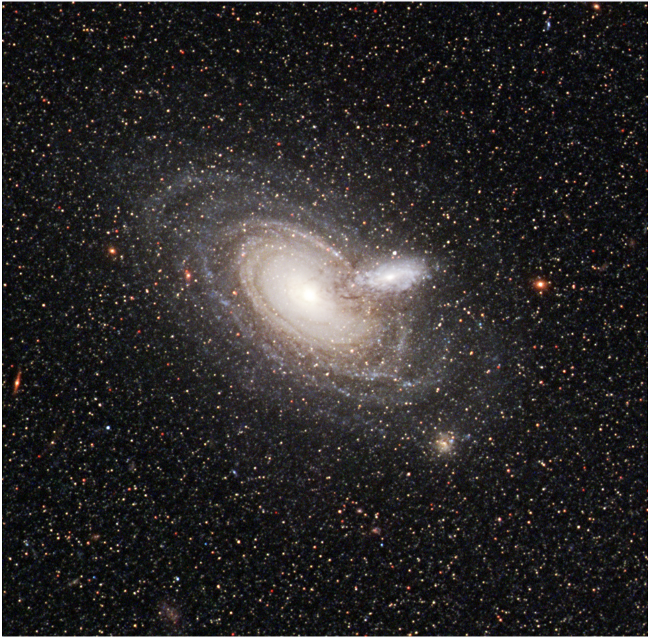
Spiral Galaxy NGC4622



Spiral Galaxy M51, the Whirlpool Galaxy



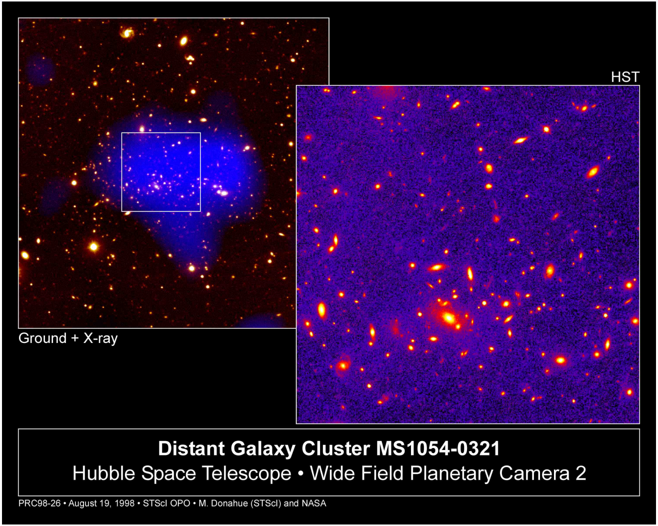
Spiral Galaxies 2MASX J00482185-2507365



Spiral Galaxy NGC3314

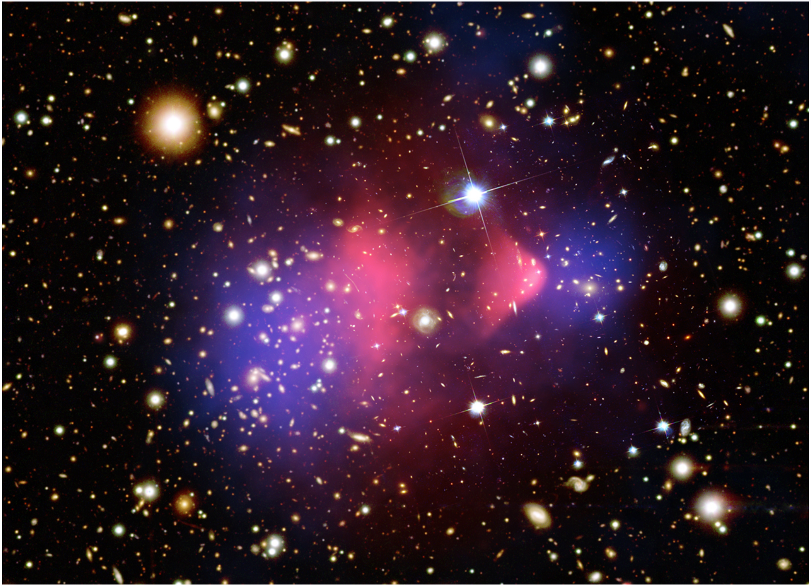
Spiral Galaxies ARP274

Galaxy Cluster MS1054-0321



The mass of galaxy clusters is roughly 5% galaxies, 10% intergalactic gas, and 85% dark matter.

The Bullet Cluster



SPIRALS

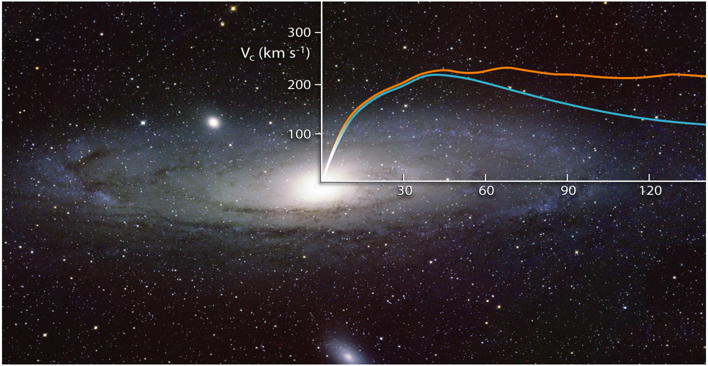
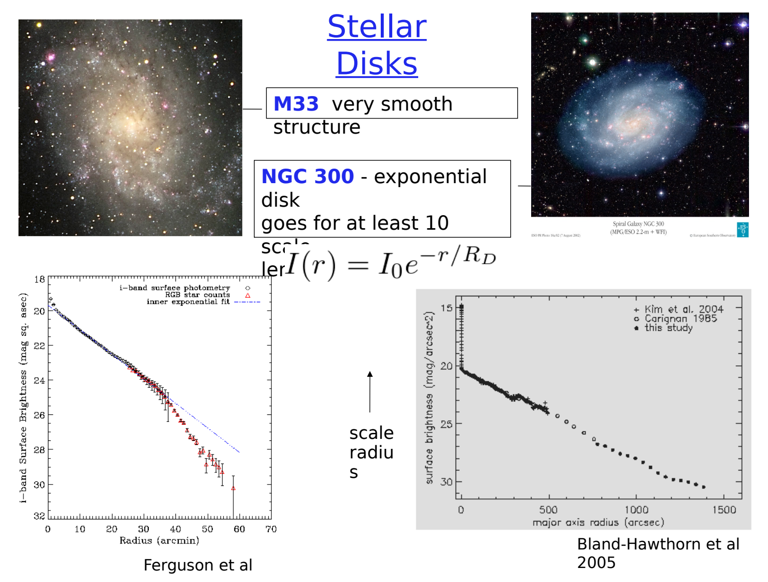


Figure: From the Dark Matter Awareness Week presentation. Presentation review at arXiv:1102.1184v1 by Paolo Salucci, Christiane Frigerio Martins, and Andrea Lapi.



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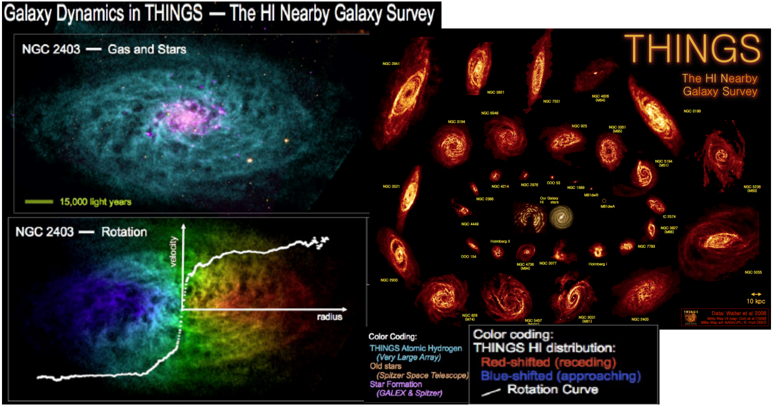
The distribution of DM around spirals

Using individual galaxies Gentile+ 2004, de Blok+ 2008 Kuzio de Naray+ 2008, Oh+ 2008, Spano+ 2008, Trachternach+ 2008, Donato+,2009

A detailed investigation: high quality data and model independent analysis



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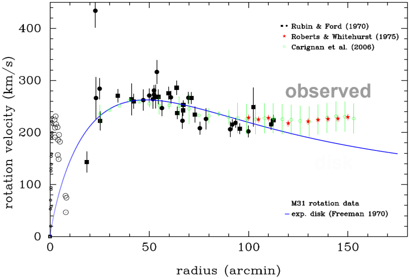
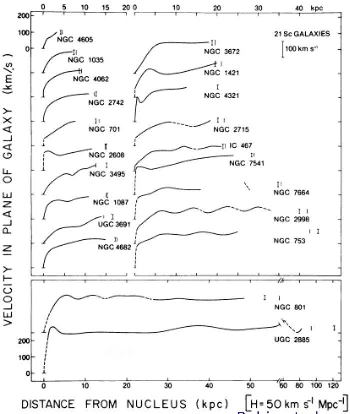
Early discovery from optical and HI RCs



no RC follows the disk velocity profile



Rubin et al 1980

Mass discrepancy AT OUTER RADII

Figure: From the Dark Matter Awareness Week presentation. Presentation review at arXiv:1102.1184v1 by Paolo Salucci, Christiane Frigerio Martins, and Andrea Lapi.

Rotation Curves

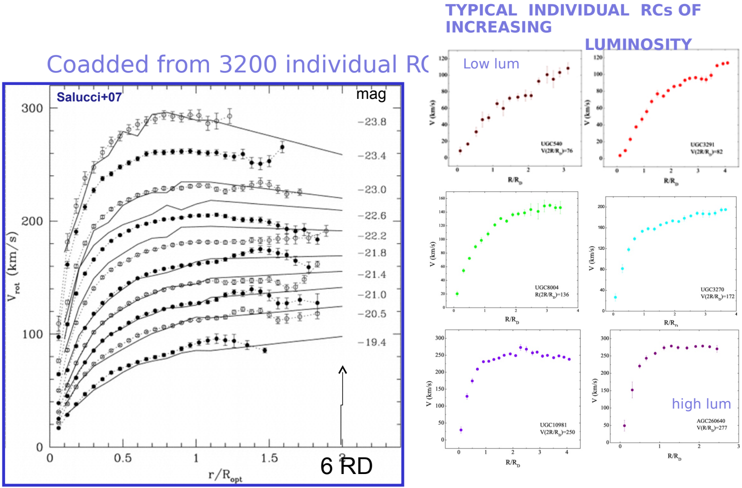


Figure: From the Dark Matter Awareness Week presentation. Presentation review at arXiv:1102.1184v1 by Paolo Salucci, Christiane Frigerio Martins, and Andrea Lapi.

The Mass of the Universe

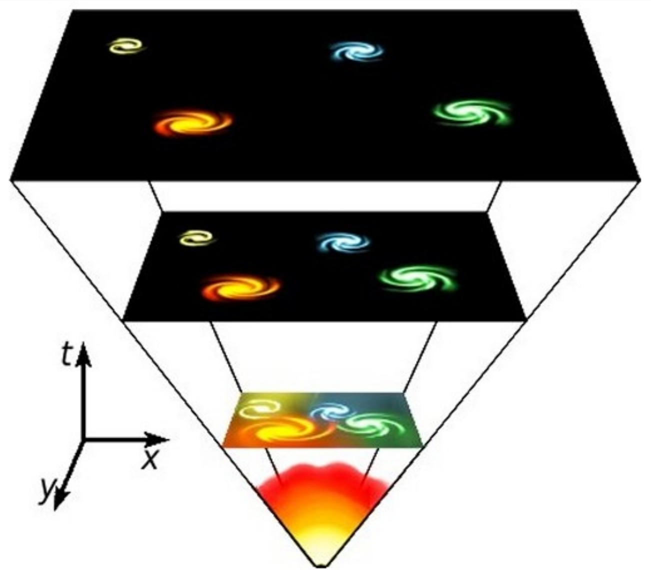
70% **Dark Energy**
(the cosmological constant of **General Relativity** used to explain the observed *accelerating* expansion of the universe)

25% **Dark Matter**

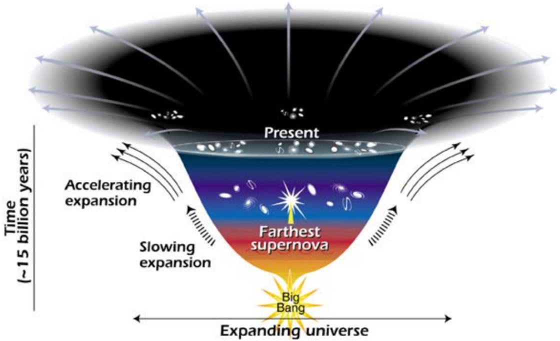
5% **Regular Baryonic Matter**
(Gas, Dust, Planets, Stars, etc., composed of particles described by **Quantum Field Theory and the Standard Model of Particle Physics**)

Which theory best describes Dark Matter?

Successes of General Relativity: The Big Bang

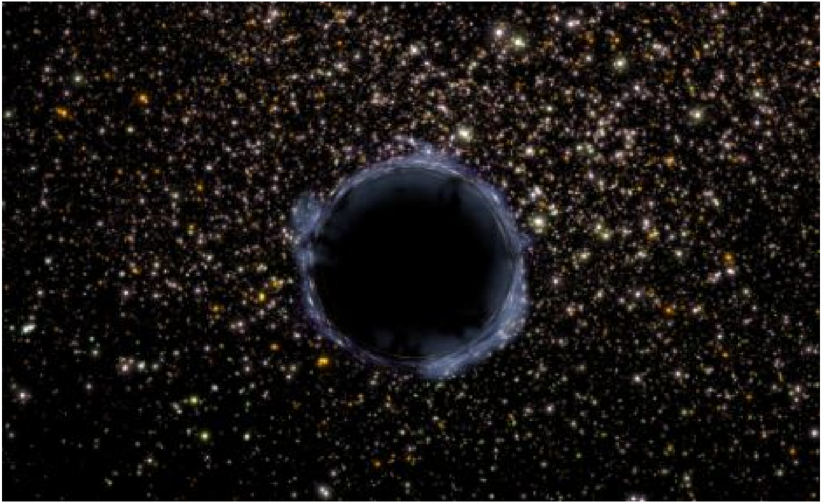


Successes of General Relativity: The *Accelerating* Expansion of the Universe



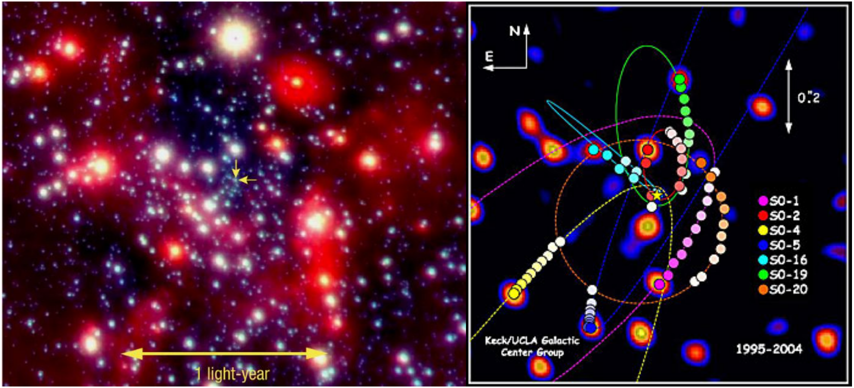
Based on papers in 1998 and 1999, the 2011 Nobel Prize in Physics was awarded to Perlmutter, Schmidt, and Riess "for the discovery of the accelerating expansion of the Universe through observations of distant supernovae."

Successes of General Relativity: Black Holes



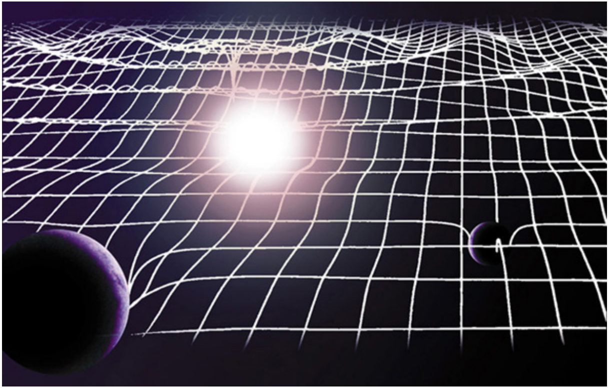
Artist's rendition of a black hole.

Successes of General Relativity: Black Holes



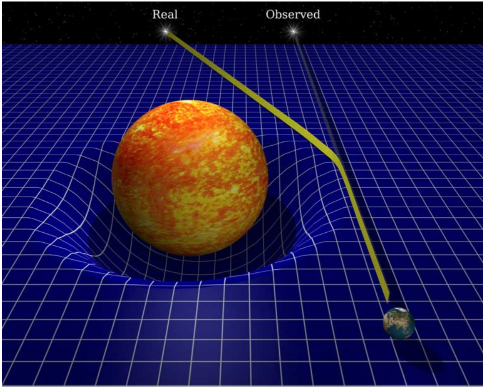
The supermassive black hole (4 million solar masses) at the center of the Milky Way Galaxy.

Successes of General Relativity: Gravity



The Earth goes around the Sun because the mass of the Sun curves spacetime, not because of some mysterious $1/r^2$ force law assumed as an axiom without any explanation as to what the mechanism for gravity might be.

Successes of General Relativity: Gravitational Lensing



General Relativity agrees with observations and predicts *twice* the bending angle for light that Newtonian physics predicts.

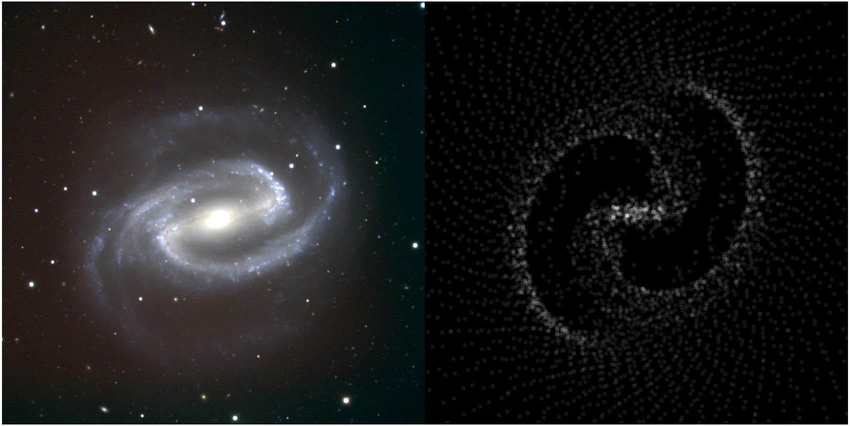
What about Dark Matter and Spiral Galaxies?

Idea 1: Natural geometric axioms motivate studying the Einstein-Klein-Gordon equations with a cosmological constant. Is the scalar field of the Klein-Gordon equation dark matter?

Idea 2: Wave types of equations, such as the Klein-Gordon equation, naturally form density waves in their matter densities.

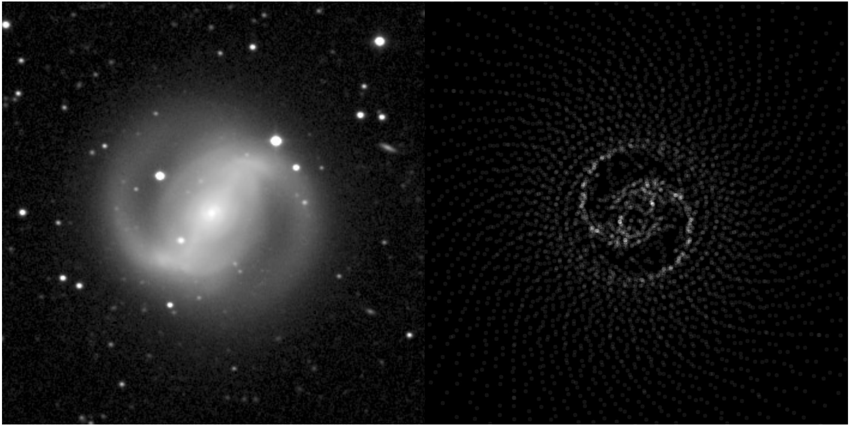
Idea 3: Density waves in dark matter, through gravity, naturally form density waves in the regular baryonic matter. Does this explain the observed spiral density waves in spiral galaxies?

Spiral Galaxy Simulation #1



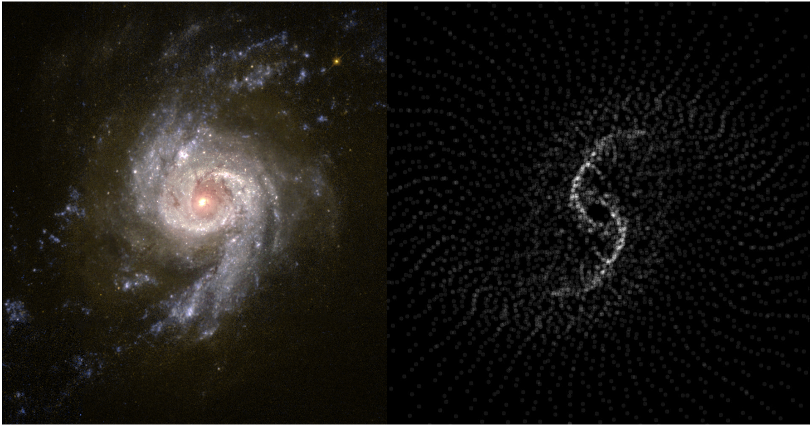
NGC1300 on the left, simulation on the right.

Spiral Galaxy Simulation #2



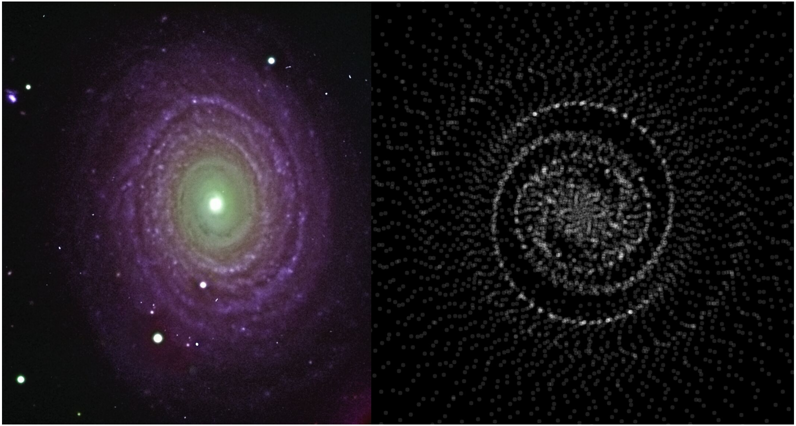
NGC4314 on the left, simulation on the right.

Spiral Galaxy Simulation #3



NGC3310 on the left, simulation on the right.

Spiral Galaxy Simulation #4



NGC488 on the left, simulation on the right.

Philosophy

General Relativity results from Special Relativity when the assumption that the spacetime metric is the standard flat one is *removed*.

The assumption that the metric is flat is replaced by the axiom that the spacetime metric is a critical point of an action functional. By Noether's theorem, spacetimes which are critical points of action functionals have conserved quantities, one for each symmetry of the action, which is great since conserved quantities like energy and momentum are fundamental observations.

Natural question:

What theory results when the assumption that the connection on the spacetime is the standard Levi-Civita one is removed? What should the action be? Even more fundamentally, what properties should the action have?

Philosophy

Axiom 0

The universe is described by a smooth manifold N *which is Hausdorff and second countable with smooth metric* g *of signature* $(-+++)$ *at every point and a smooth connection* ∇*.*

A smooth manifold N is a Hausdorff space with a complete atlas of smoothly overlapping coordinate charts. Hence, we see that coordinate charts are more than convenient places to do calculations, but are in fact a necessary part of the definition of a smooth manifold.

Philosophy

Given a fixed coordinate chart, let $\{\partial_i\}$, $0 \leqslant i \leqslant 3$, be the tangent vector fields to N corresponding to the standard basis vector fields of the coordinate chart.

Let $g_{ij} = g(\partial_i, \partial_j)$ and $\Gamma_{ijk} = g(\nabla_{\partial_i}\partial_j, \partial_k)$, and let

$$M = \{g_{ij}\} \quad , \quad C = \{\Gamma_{ijk}\} \quad , \quad M' = \{g_{ij,k}\} \quad \text{and} \quad C' = \{\Gamma_{ijk,l}\}$$

be the components of the metric and the connection in the coordinate chart and all of the first derivatives of these components in the coordinate chart.

Philosophy

Axiom 1

For all coordinate charts $\Phi : \Omega \subset N \to R^4$ *and open sets* U *whose closure is compact and in the interior of* Ω*,* (g, ∇) *is a critical point of the functional*

$$F_{\Phi,U}(g,\nabla) = \int_{\Phi(U)} \mathit{Quad}_M(M' \cup M \cup C' \cup C)\, dV_{R^4}$$

with respect to smooth variations of the metric and connection compactly supported in U*, for some fixed quadratic functional* $Quad_M$ *with coefficients in* M*, where we define*

$$\mathit{Quad}_Y(\{x_\alpha\}) = \sum_{\alpha,\beta} F^{\alpha\beta}(Y) x_\alpha x_\beta$$

for some fixed functions $\{F^{\alpha\beta}\}$*.*

Philosophy

Note that we have not arbitrarily specified the action, only the form of the action. Also note that while there is one action for each coordinate chart, (g, ∇) must be a critical point of these actions in *all* coordinate charts and hence does not depend on any one particular coordinate chart.

$\text{Quad}_M(M')$ $\longrightarrow$ Vacuum Einstein Equation

$\text{Quad}_M(M' \cup M)$ $\longrightarrow$ Vacuum Einstein Equation with a Cosmological Constant

$\text{Quad}_M(M' \cup M \cup C' \cup C)$ $\longrightarrow$ Einstein-Klein-Gordon Equations with a Cosmological Constant

When the integrand in Axiom 1 is replaced with the above expressions, we get the corresponding three systems of equations on the right. The first two statements follow from the works of Cartan, Weyl, Vermeil, and Lovelock. The last statement is what we will now discuss.

The Einstein-Hilbert Action

Standard calculations show that the formula for the scalar curvature in terms of the metric in a coordinate chart is

$$
\begin{aligned}
R \;=\; & (g^{ik}g^{jl} - g^{ij}g^{kl})\underline{g_{ij,kl}} + \underline{g_{ij,k}g_{ab,c}} \cdot \\
& \left(\frac{3}{4}g^{ia}g^{jb}g^{kc} - \frac{1}{2}g^{ia}g^{jc}g^{kb} - g^{ia}g^{jk}g^{bc} - \frac{1}{4}g^{ij}g^{ab}g^{kc} + g^{ij}g^{ac}g^{kb}\right)
\end{aligned}
$$

Then since $dV = |g|^{1/2}\, dV_{R^4}$, integrating by parts gives

$$
\begin{aligned}
\int_U R\, dV \;=\; & \text{boundary term } + \int_{\Phi(U)} \underline{g_{ij,k}g_{ab,c}} \cdot |g|^{1/2}\, dV_{R^4} \cdot \\
& \left(-\frac{1}{4}g^{ia}g^{jb}g^{kc} + \frac{1}{2}g^{ia}g^{jc}g^{kb} + \frac{1}{4}g^{ij}g^{ab}g^{kc} - \frac{1}{2}g^{ij}g^{ac}g^{kb}\right)
\end{aligned}
$$

The Einstein-Hilbert action fits the form of Axiom 1 with no connection terms. This is why the resulting Euler-Lagrange equation, $G = 0$, is second order in the metric.

The General Form of a Connection

By the Koszul formula, the standard Levi-Civita connection has components

$$\bar{\Gamma}_{ijk} = \frac{1}{2}\left(g_{ik,j} + g_{jk,i} - g_{ij,k}\right),$$

The difference of two connections is a tensor, so let

$$D_{ijk} = \Gamma_{ijk} - \bar{\Gamma}_{ijk}.$$

Define

$$\begin{aligned} T_{ijk} &= D_{ijk} - D_{jik} \\ &= (\Gamma_{ijk} - \bar{\Gamma}_{ijk}) - (\Gamma_{jik} - \bar{\Gamma}_{jik}) \\ &= \Gamma_{ijk} - \Gamma_{jik} \end{aligned}$$

which we recognize as the components of the torsion tensor. Note that T_{ijk}, unlike D_{ijk}, does *not* depend on derivatives of the metric.

Define

$$\begin{aligned}\gamma_{ijk} &= \frac{1}{6}(T_{ijk}+T_{jki}+T_{kij})\\ &= \frac{1}{6}(D_{ijk}-D_{jik}+D_{jki}-D_{kji}+D_{kij}-D_{ikj})\\ &= \frac{1}{6}(\Gamma_{ijk}-\Gamma_{jik}+\Gamma_{jki}-\Gamma_{kji}+\Gamma_{kij}-\Gamma_{ikj})\end{aligned}$$

to be the fully antisymmetric part of the difference tensor D. Thus, γ_{ijk} are the components of a three form. Hence,

$$d\gamma_{ijkl} = \gamma_{jkl,i} - \gamma_{kli,j} + \gamma_{lij,k} - \gamma_{ijk,l}$$

are the antisymmetric coefficients of the tensor $d\gamma$ which do *not* involve derivatives of the metric, just derivatives of Γ. Hence, functionals of the form

$$F_{\Phi,U}(g,\nabla) = \int_U (cR - 2\Lambda - \frac{c_3}{24}|d\gamma|^2 - \mathsf{Quad}_g(D))\, dV,$$

are allowed by Axiom 1, up to a boundary term which is irrelevant for the Euler-Lagrange equations produced. Conjecture: this is it.

The Action

In the simplest representative case, we can choose $D_{ijk} = \gamma_{ijk}$ with action functional

$$\begin{aligned} F_{\Phi,U}(g,\nabla) &= \int_U (R - 2\Lambda - \frac{c_3}{24}|d\gamma|^2 - \frac{c_4}{6}|\gamma|^2)\, dV \\ &= \int_U (R - 2\Lambda - c_3|d\gamma|^2_{4-form} - c_4|\gamma|^2_{3-form})\, dV \end{aligned}$$

Equivalently, if we let

$$\gamma = *(v^*),$$

where v is a vector field, v^* is the 1 form dual to v, and $*$ is the Hodge star operator, then the action becomes

$$F_{\Phi,U}(g,\nabla) = \int_U (R - 2\Lambda + c_3(\nabla \cdot v)^2 + c_4|v|^2)\, dV,$$

where $\nabla \cdot v$ denotes the divergence of v.

The Euler-Lagrange equations for this action are

$$
\begin{aligned}
G + \Lambda g &= c_4\, v^* \otimes v^* - \frac{1}{2}\left(c_3(\nabla \cdot v)^2 + c_4|v|^2\right) g \\
\nabla(\nabla \cdot v) &= \frac{c_4}{c_3} v.
\end{aligned}
$$

For the dominant energy condition to be satisfied, we need $c_3, c_4 \geqslant 0$. To arrive at a nontrivial equation for v we need $c_3 \neq 0$ and to arrive at a deterministic equation for v we need $c_4 \neq 0$. Hence, let's take $c_3, c_4 > 0$. Now let

$$
f = \left(\frac{c_3}{c_4}\right)^{1/2} \nabla \cdot v \qquad \Rightarrow \qquad v = \left(\frac{c_3}{c_4}\right)^{1/2} \nabla f
$$

and

$$
\begin{aligned}
G + \Lambda g &= c_3 \left\{ df \otimes df - \frac{1}{2}\left(|df|^2 + \frac{c_4}{c_3} f^2\right) g \right\} \\
\Box_g f &= \frac{c_4}{c_3} f
\end{aligned}
$$

which has a solution if and only if the original system does.

The Einstein-Klein-Gordon Equations

$$
\begin{aligned}
G + \Lambda g &= 8\pi\mu_0 \left\{ 2\frac{df \otimes df}{\Upsilon^2} - \left(\frac{|df|^2}{\Upsilon^2} + f^2 \right) g \right\} \\
\Box_g f &= \Upsilon^2 f
\end{aligned}
$$

where G is the Einstein curvature tensor, f is the scalar field representing dark matter, Λ is the cosmological constant, and Υ is a new fundamental constant of nature whose value has yet to be determined. Note that the connection will have components

$$
\Gamma_{ijk} = \frac{1}{\Upsilon}(*df)_{ijk} + \frac{1}{2}\left(g_{ik,j} + g_{jk,i} - g_{ij,k}\right).
$$

Deep question: The effect of the connection is seen gravitationally as the scalar field f, but does the connection manifest itself physically in any other way?

Wave Dark Matter is Automatically Cold

Suppose that the spacetime metric is both homogeneous and isotropic, and hence is the Friedmann-Lemaître-Robertson-Walker metric $-dt^2 + a(t)^2 ds_\kappa^2$, where ds_κ^2 is the constant curvature metric of curvature κ. Then f is solely a function of t.

Furthermore, if we let $H(t) = a'(t)/a(t)$ be the Hubble constant, and $\bar{\rho}$ and $\bar{P}$ be the average energy density and average pressure of the scalar field for $a \leqslant t \leqslant b$, then

$$\frac{\bar{P}}{\bar{\rho}} = \frac{\epsilon}{1+\epsilon} \qquad \text{where} \qquad \epsilon = -\frac{3\overline{H'}}{4\Upsilon^2}$$

and

$$\overline{H'} = \frac{\int_a^b H'(t) f(t)^2 \, dt}{\int_a^b f(t)^2 \, dt},$$

where a, b are two zeros of f (for example, two consecutive zeros).

$|H'(t)| \approx (10^{10} \text{ light years})^{-2}$. Could the average temperature of dark matter in the universe be used to estimate the value of Υ?

Solutions to the Klein-Gordon Equation on Static Spherically Symmetric Spacetimes

Suppose the spacetime metric has the form

$$ds^2 = -V(r)^2 dt^2 + V(r)^{-2} \left(dx^2 + dy^2 + dz^2\right)$$

The function $V(r)$ acts likes the gravitational potential function from Newtonian physics but goes to one at infinity.

Then

$$f = A\cos(\omega t) \cdot Y_n(\theta, \phi) \cdot r^n \cdot f_{\omega,n}(r)$$

is a solution to the Klein-Gordon equation of this spacetime when

$$V(r)^2 \left(f''_{\omega,n}(r) + \frac{2(n+1)}{r} f'_{\omega,n}(r) \right) = \left(\Upsilon^2 - \frac{\omega^2}{V(r)^2} \right) f_{\omega,n}.$$

Rotating Wave Dark Matter Solutions

The solutions on which our simulations are based are of the form

$$f = A_0 \cos(\omega_0 t) f_{\omega_0,0}(r) + A_2 \cos(\omega_2 t - 2\phi) \sin^2(\theta) r^2 f_{\omega_2,2}(r).$$

Note that both $\cos(2\phi)\sin^2(\theta)$ and $\sin(2\phi)\sin^2(\theta)$ are second degree spherical harmonics, so this fits the previous form.

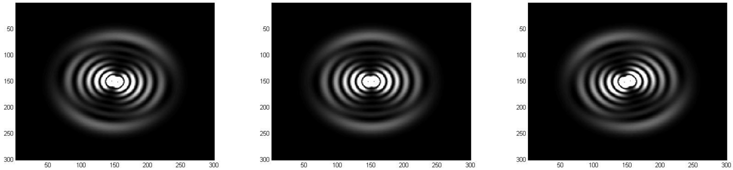


Figure: Exact solution to the Klein-Gordon equation in a fixed spherically symmetric potential well based on the Milky Way Galaxy at $t = 0$, $t = 10$ million years, and $t = 20$ million years. The pictures show the dark matter density (in white) in the xy plane. This solution, which one can see is rotating, has angular momentum.

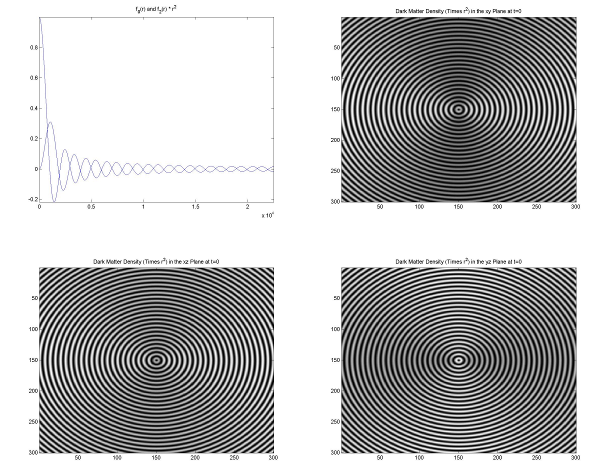


Figure: Spiral Galaxy Simulation # 2: Graphs of $f_{\omega_0,0}(r)$ and $r^2 f_{\omega_2,2}(r)$ for r up to $22,500$ light years (top left). The other three images, each with a radius of $22,500$ light years, are plots of the dark matter density (in white) times r^2 in the xy plane (top right), in the xz plane (bottom left), and in the yz plane (bottom right).

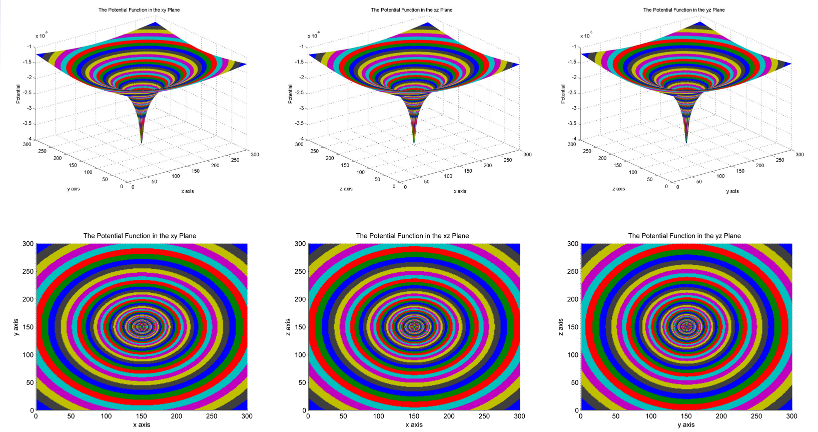


Figure: Graphs of the potential function in the xy plane (first column), the xz plane (second column), and the yz plane (third column) out to a radius of $22,500$ light years for Spiral Galaxy Simulation #2. The second row is the same as the first row except that the point of view is looking straight down so that we can see the level sets of the potential function in each plane. Note that the level sets are slightly ellipsoidal.

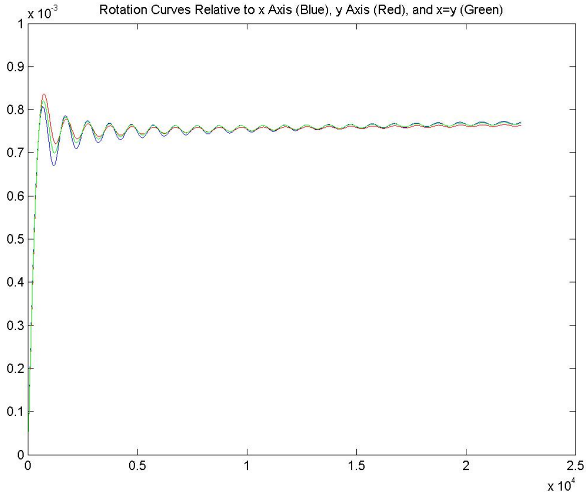


Figure: Approximate rotation curves for Spiral Galaxy Simulation #2. We have approximated the rotation curves with graphs of $\sqrt{r|\nabla V|}$ (which is exactly correct in the spherically symmetric case) along the x axis (in blue), along the y axis (in red), and along $y = x$ (in green).

Spiral Galaxy Simulation #2

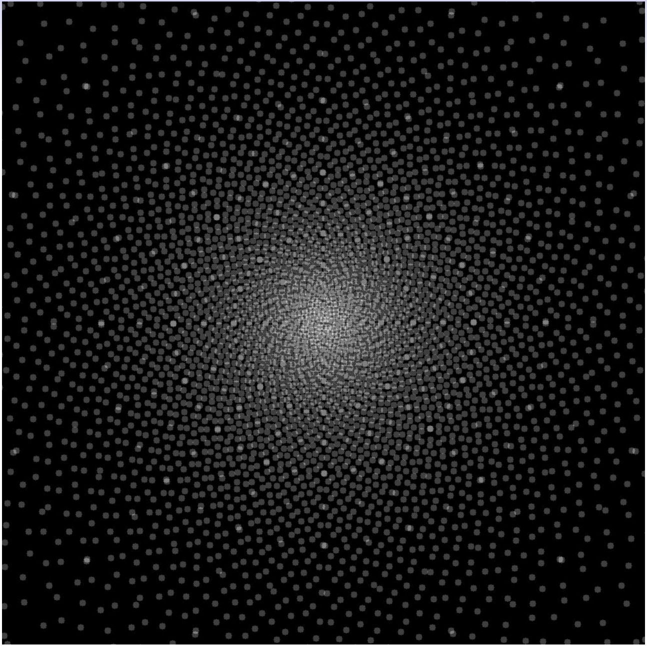


Figure: $t = 0$ million years for Spiral Galaxy Simulation #2.

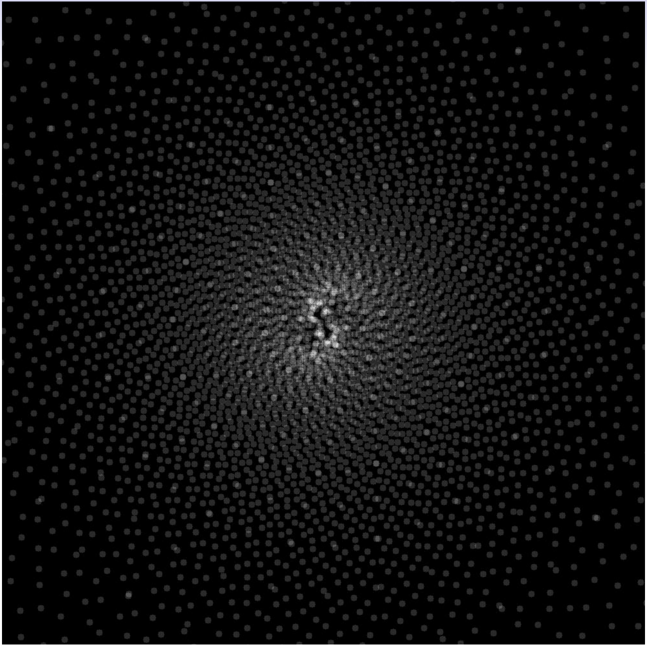


Figure: $t = 5$ million years for Spiral Galaxy Simulation #2.

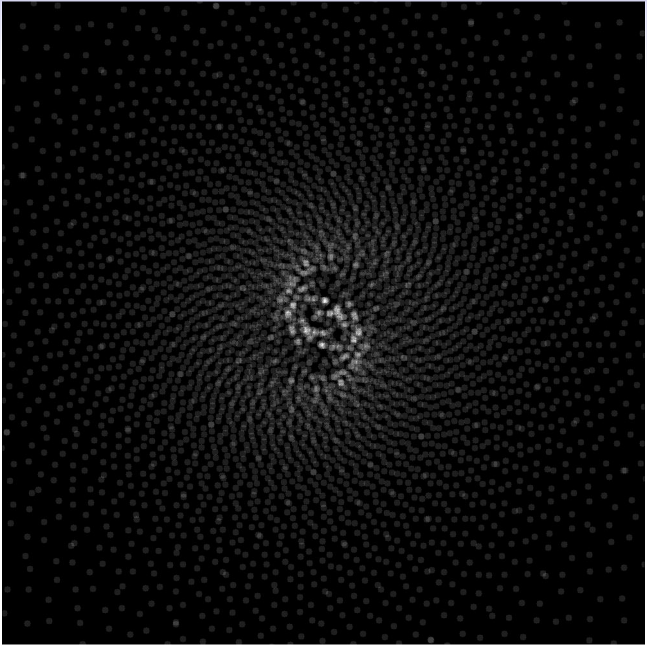


Figure: $t = 10$ million years for Spiral Galaxy Simulation #2.

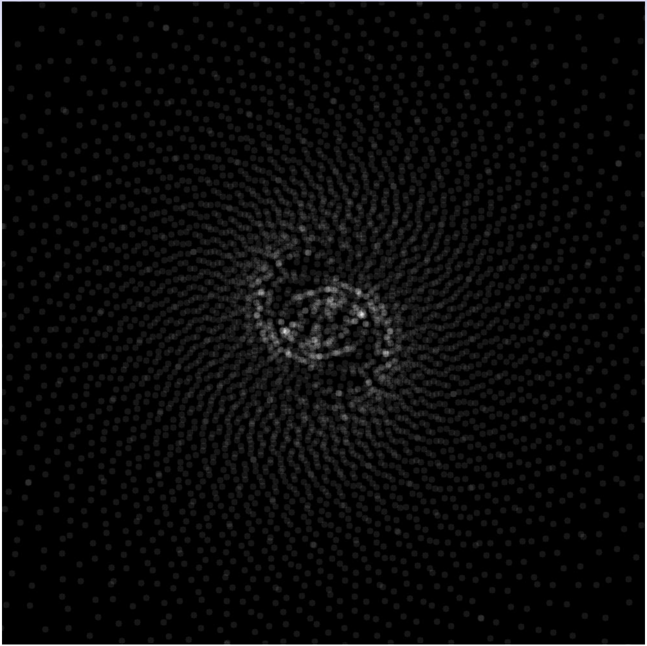


Figure: $t = 15$ million years for Spiral Galaxy Simulation $\#2$.

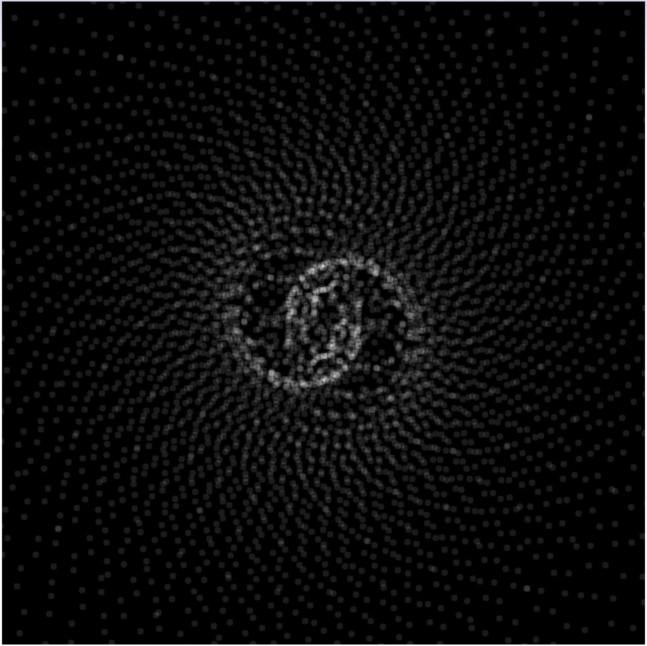


Figure: $t = 20$ million years for Spiral Galaxy Simulation #2.

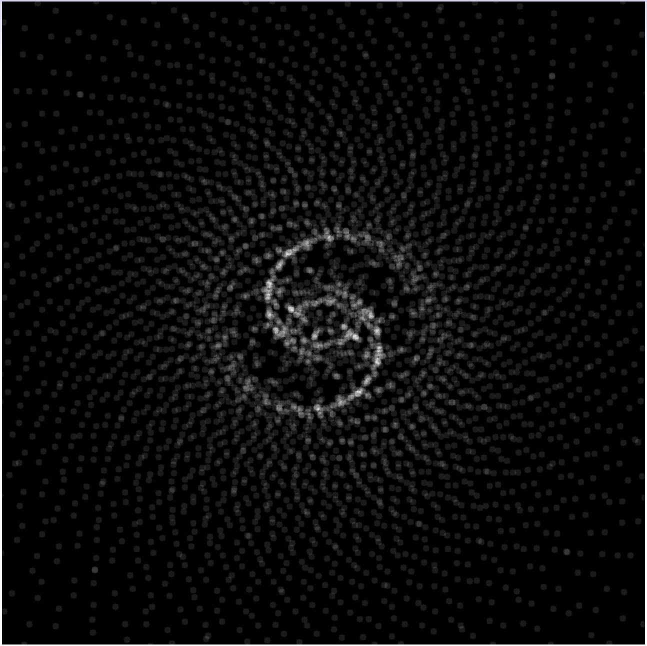


Figure: $t = 25$ million years for Spiral Galaxy Simulation #2.

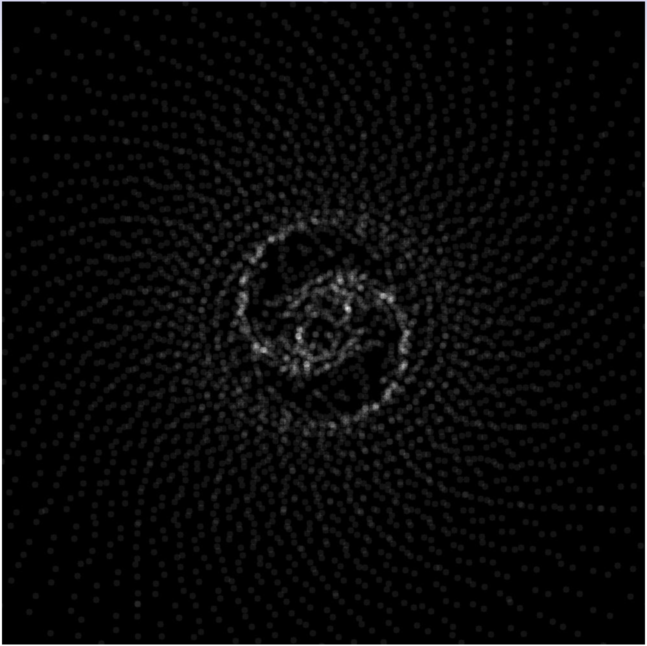


Figure: $t = 30$ million years for Spiral Galaxy Simulation #2.

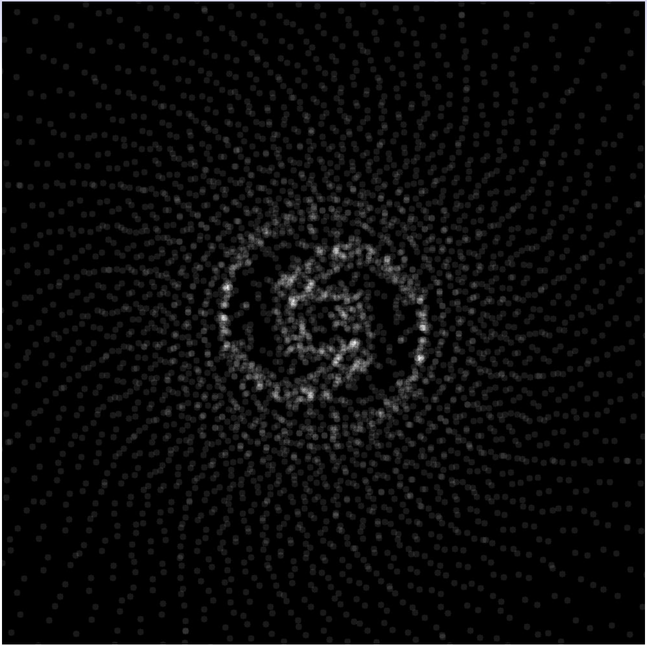


Figure: $t = 35$ million years for Spiral Galaxy Simulation #2.

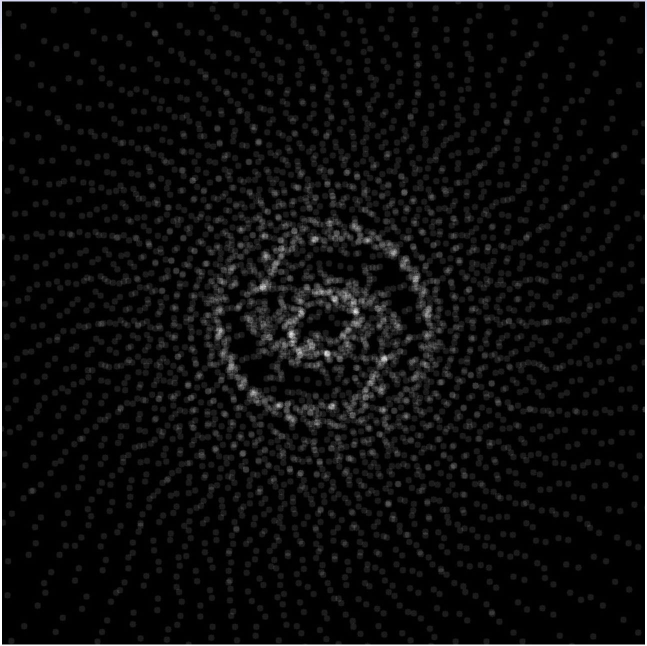


Figure: $t = 40$ million years for Spiral Galaxy Simulation #2.

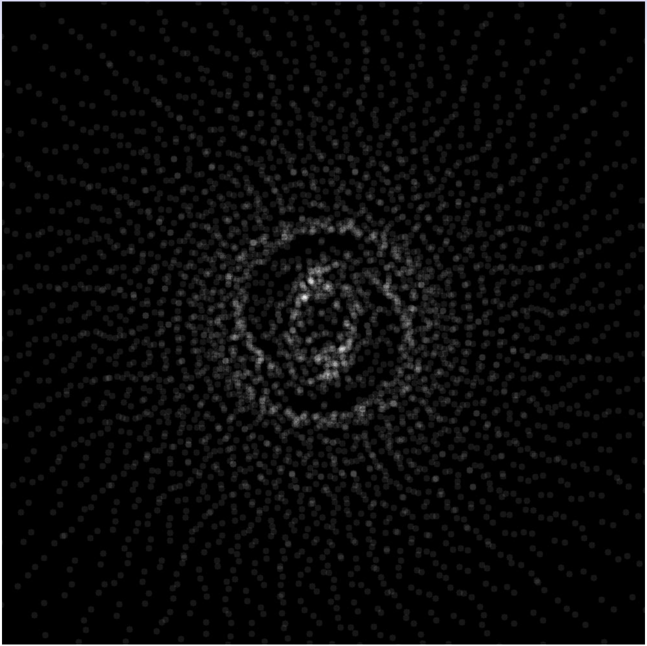


Figure: $t = 45$ million years for Spiral Galaxy Simulation #2.

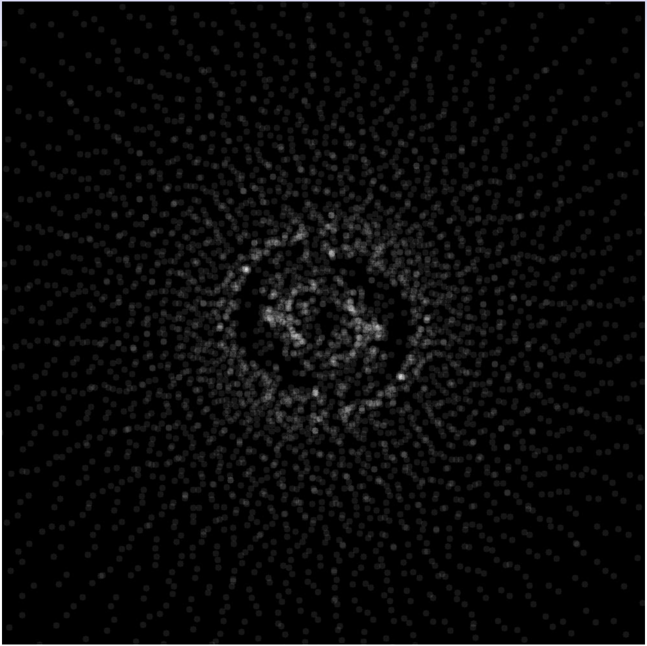
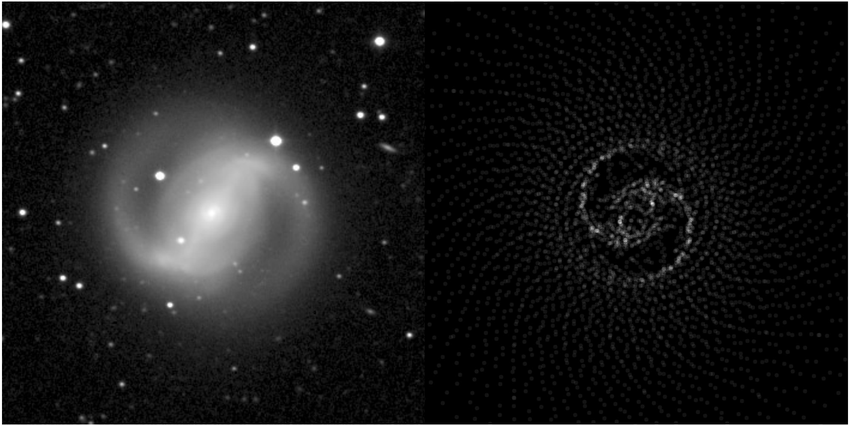


Figure: $t = 50$ million years for Spiral Galaxy Simulation #2.

Spiral Galaxy Simulation #2



NGC4314 on the left, simulation on the right.

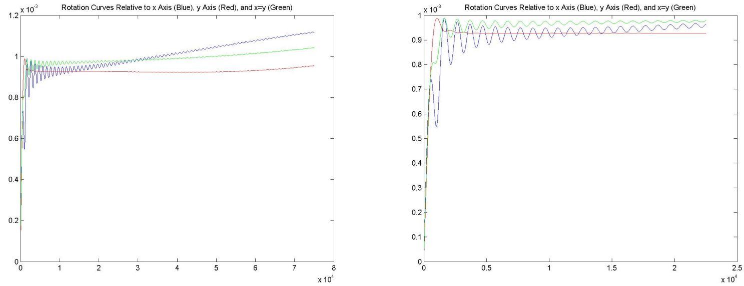


Figure: Approximate rotation curves for Spiral Galaxy Simulation #1 out to a radius of $75,000$ light years (left) and $22,500$ light years (right). We have approximated the rotation curves with graphs of $\sqrt{r|\nabla V|}$ (which is exactly correct in the spherically symmetric case) along the x axis (in blue), along the y axis (in red), and along $y = x$ (in green).

Spiral Galaxy Simulation #1

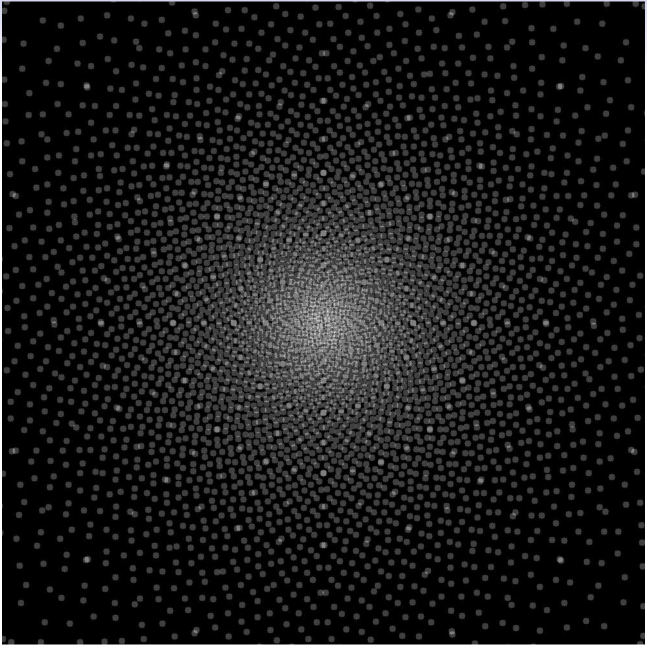


Figure: $t = 0$ million years for Spiral Galaxy Simulation #1.

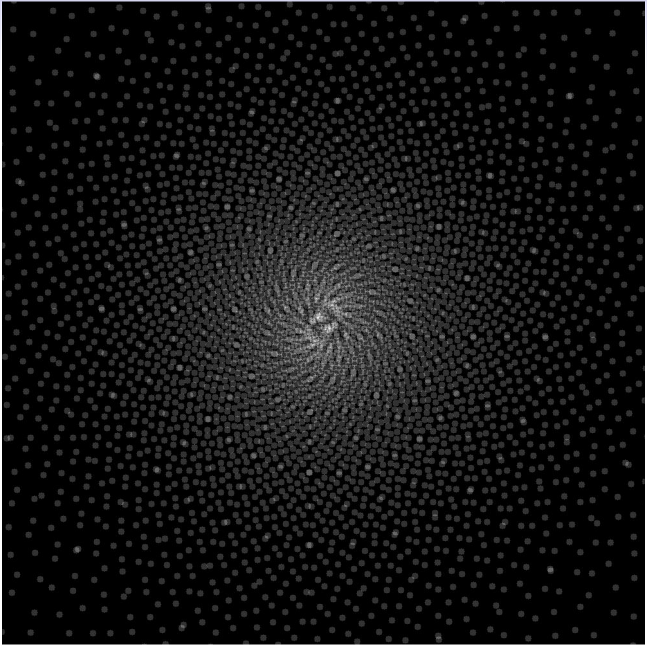


Figure: $t = 1$ million years for Spiral Galaxy Simulation #1.

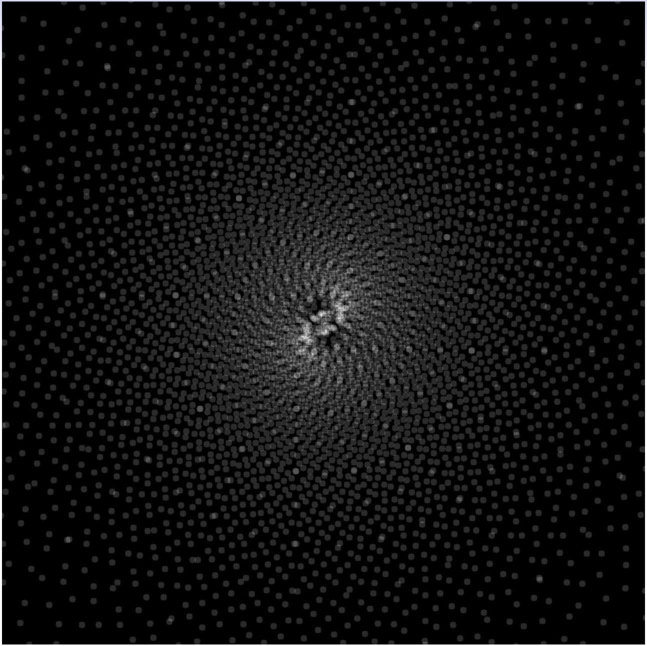


Figure: $t = 2$ million years for Spiral Galaxy Simulation #1.

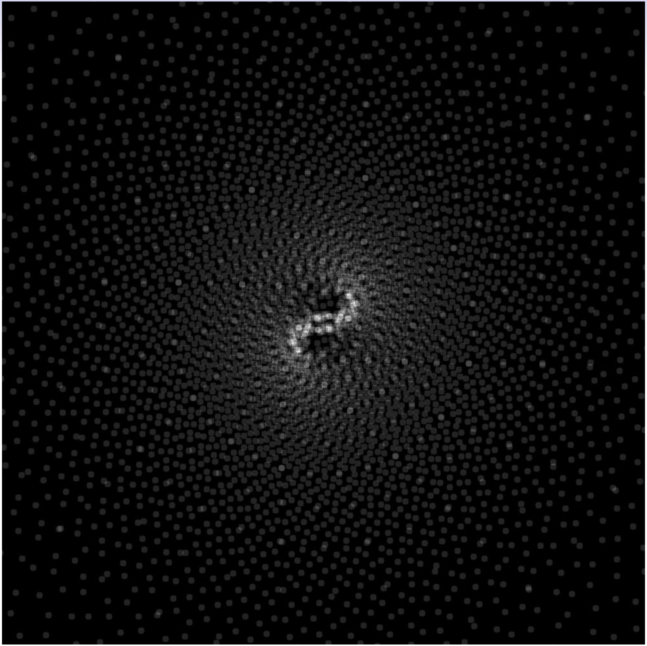


Figure: $t = 3$ million years for Spiral Galaxy Simulation #1.

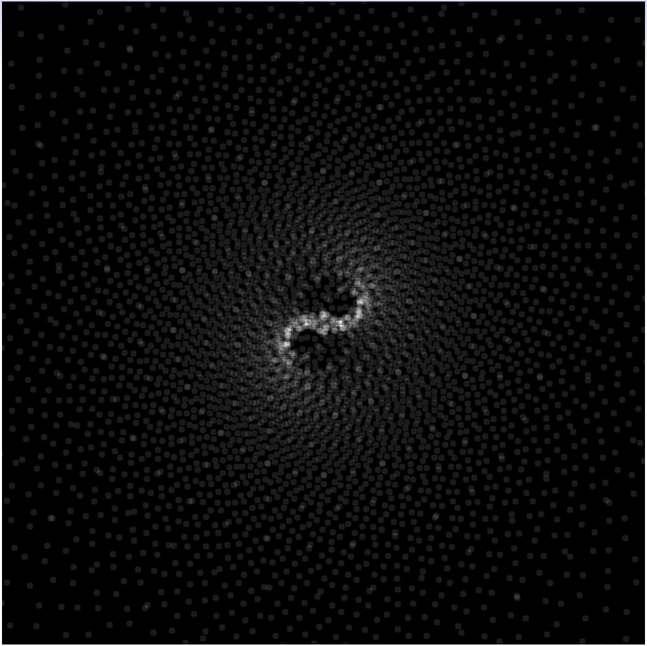


Figure: $t = 4$ million years for Spiral Galaxy Simulation $\#1$.

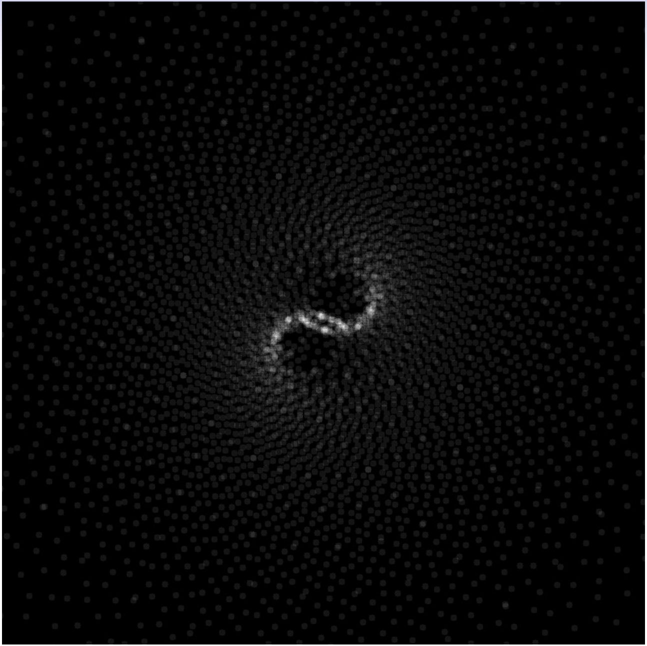


Figure: $t = 5$ million years for Spiral Galaxy Simulation #1.

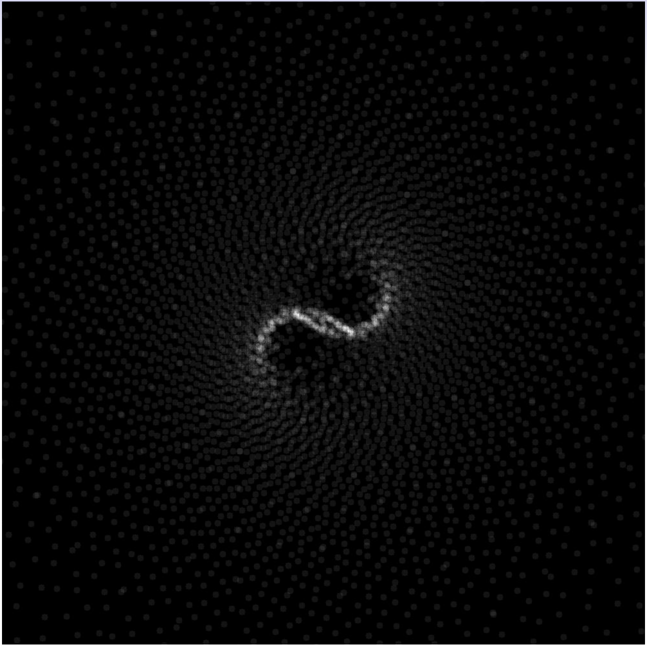


Figure: $t = 6$ million years for Spiral Galaxy Simulation #1.

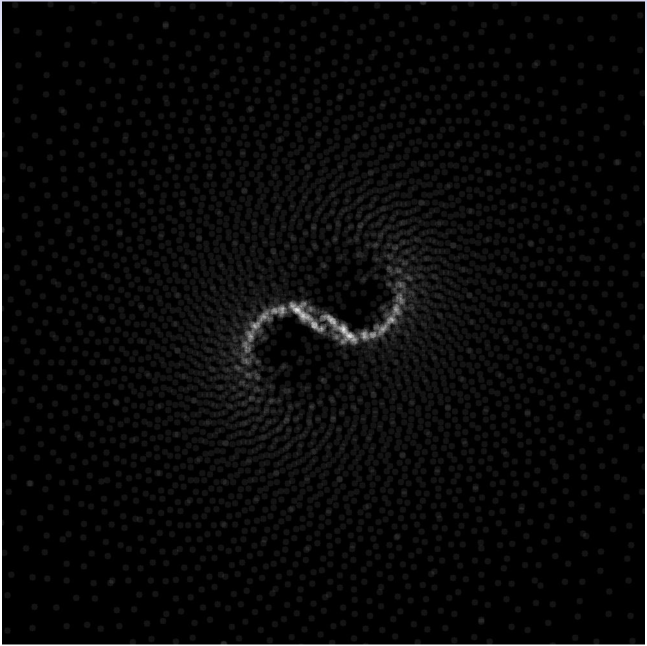


Figure: $t = 7$ million years for Spiral Galaxy Simulation #1.

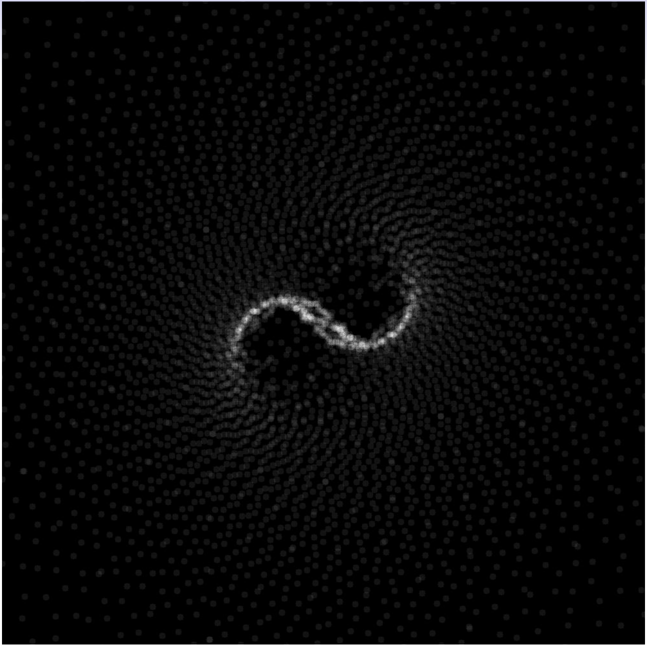


Figure: $t = 8$ million years for Spiral Galaxy Simulation #1.

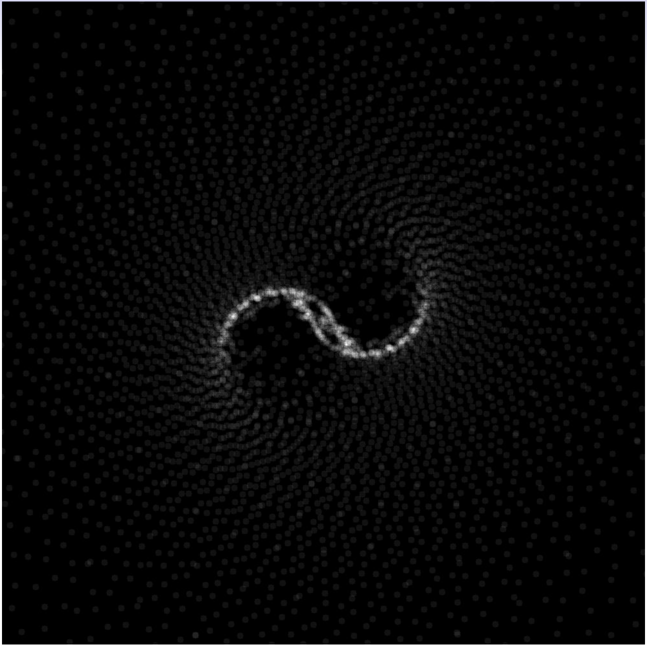


Figure: $t = 9$ million years for Spiral Galaxy Simulation #1.

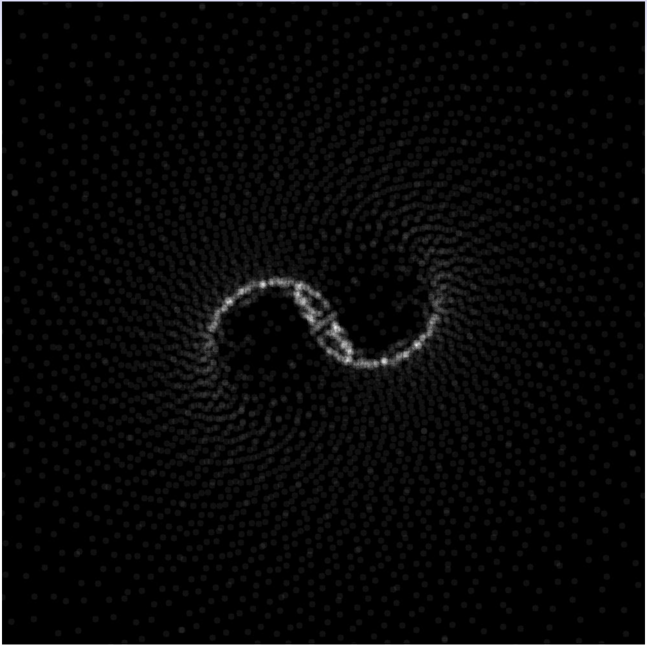


Figure: $t = 10$ million years for Spiral Galaxy Simulation $\#1$.

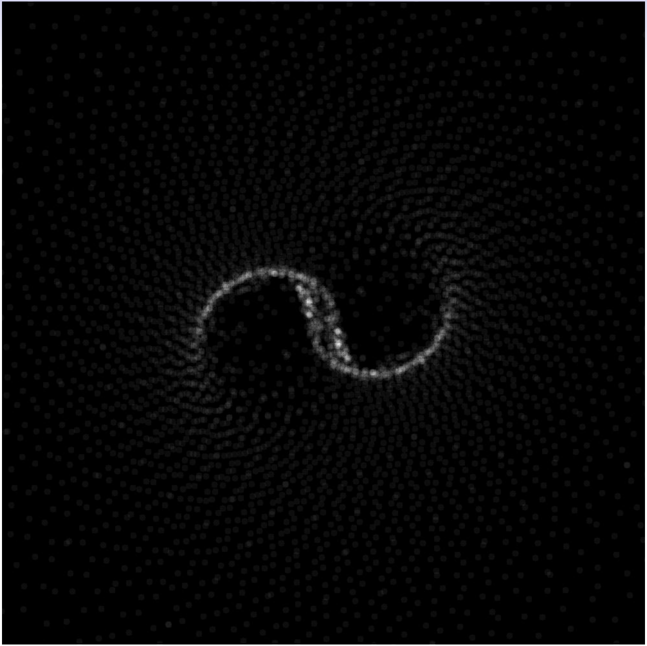


Figure: $t = 11$ million years for Spiral Galaxy Simulation #1.

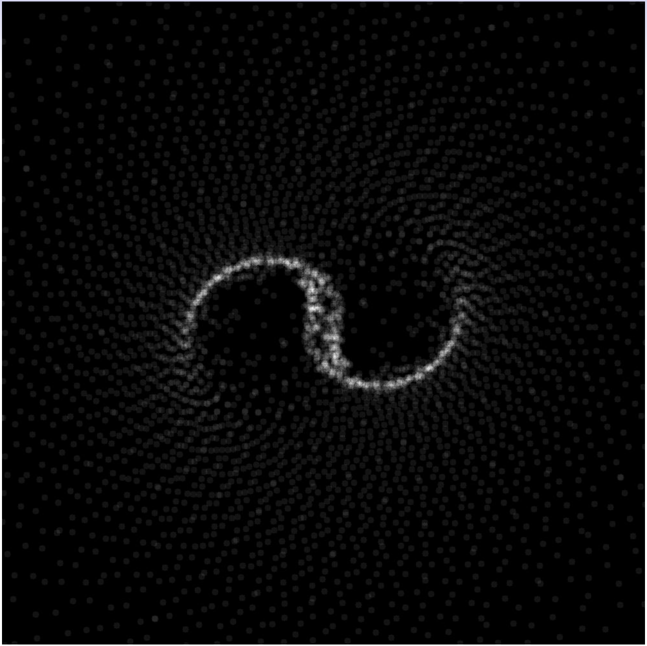


Figure: $t = 12$ million years for Spiral Galaxy Simulation #1.

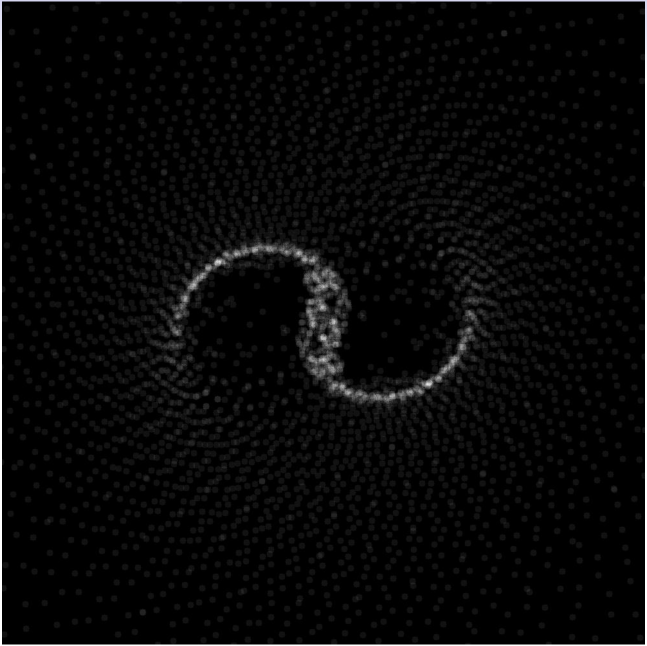


Figure: $t = 13$ million years for Spiral Galaxy Simulation $\#1$.

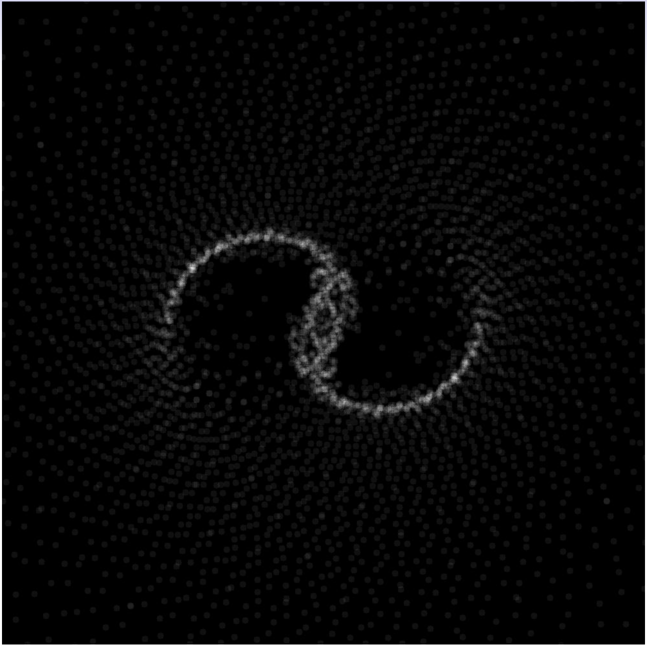


Figure: $t = 14$ million years for Spiral Galaxy Simulation #1.

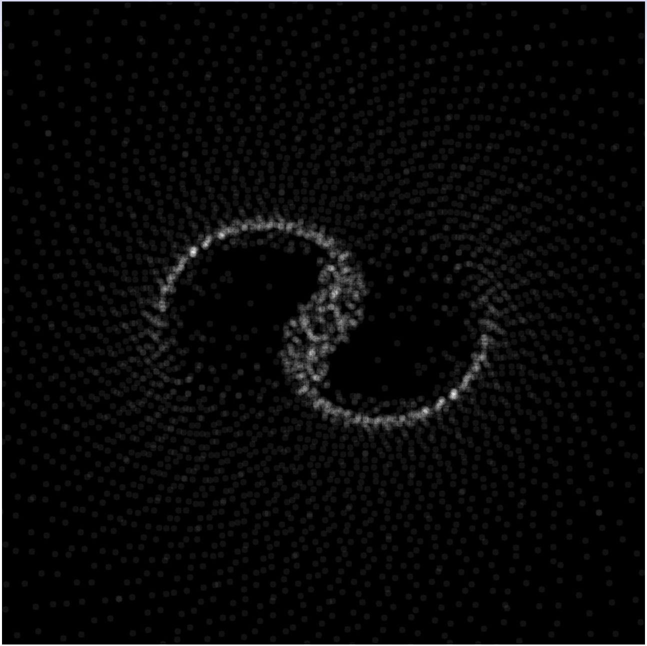


Figure: $t = 15$ million years for Spiral Galaxy Simulation #1.

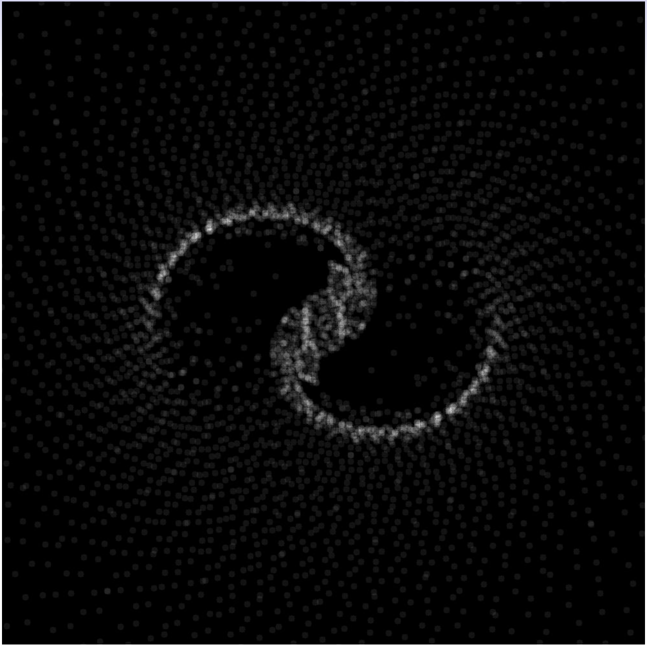


Figure: $t = 16$ million years for Spiral Galaxy Simulation $\#1$.

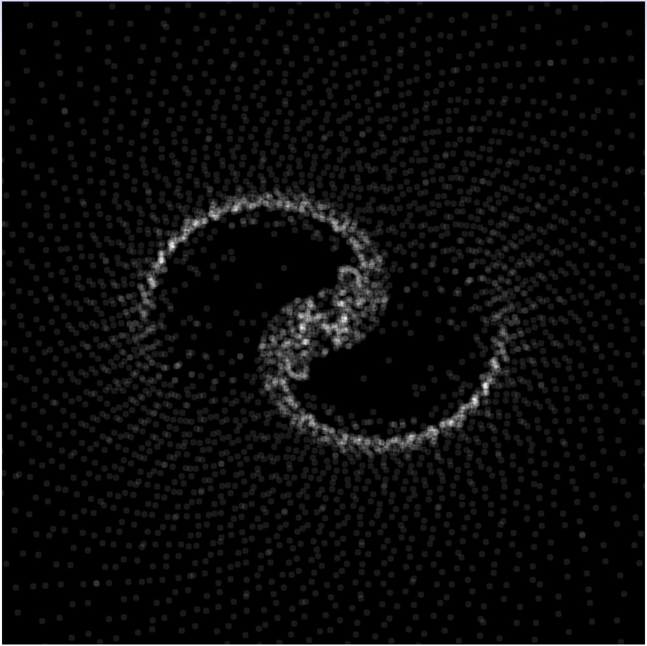


Figure: $t = 17$ million years for Spiral Galaxy Simulation #1.

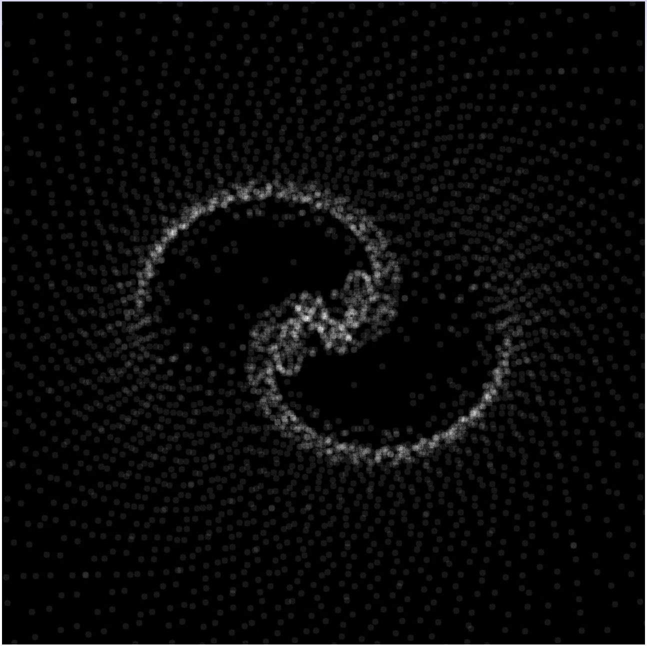


Figure: $t = 18$ million years for Spiral Galaxy Simulation #1.

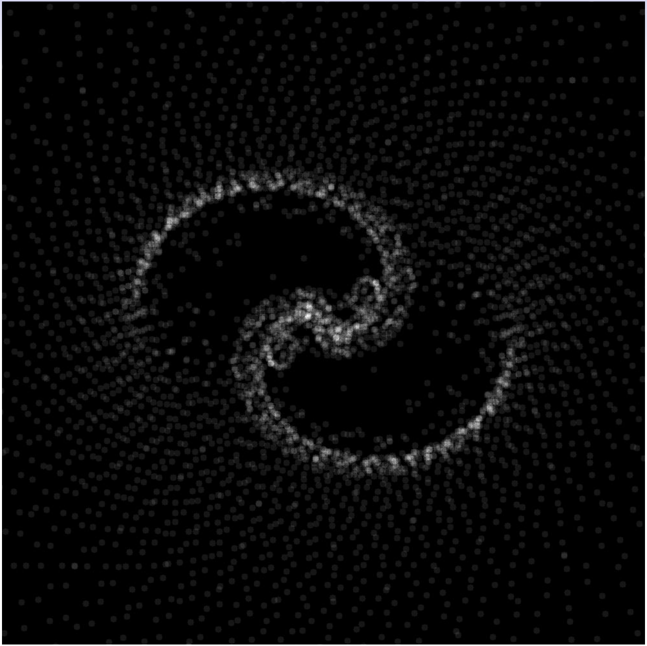


Figure: $t = 19$ million years for Spiral Galaxy Simulation $\#1$.

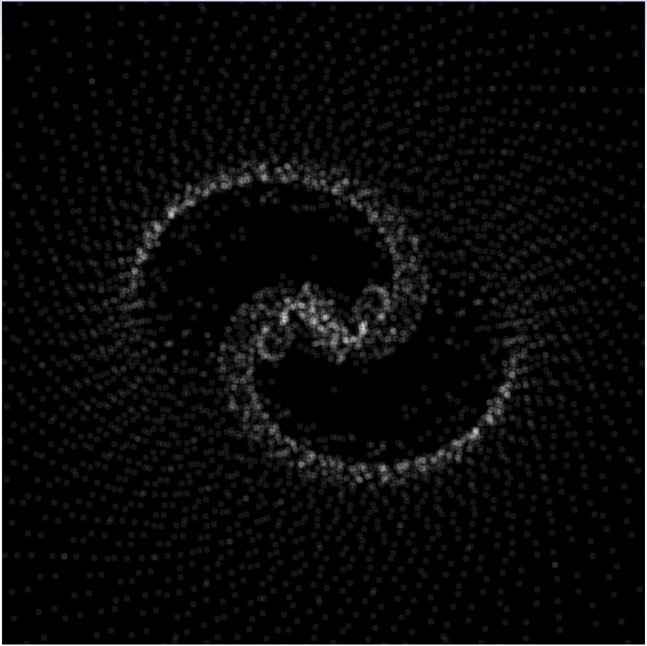


Figure: $t = 20$ million years for Spiral Galaxy Simulation #1.

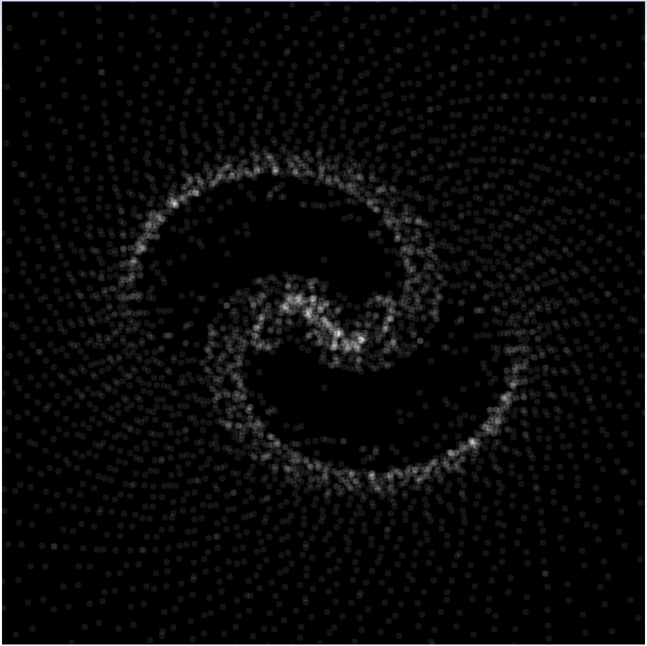


Figure: $t = 21$ million years for Spiral Galaxy Simulation #1.

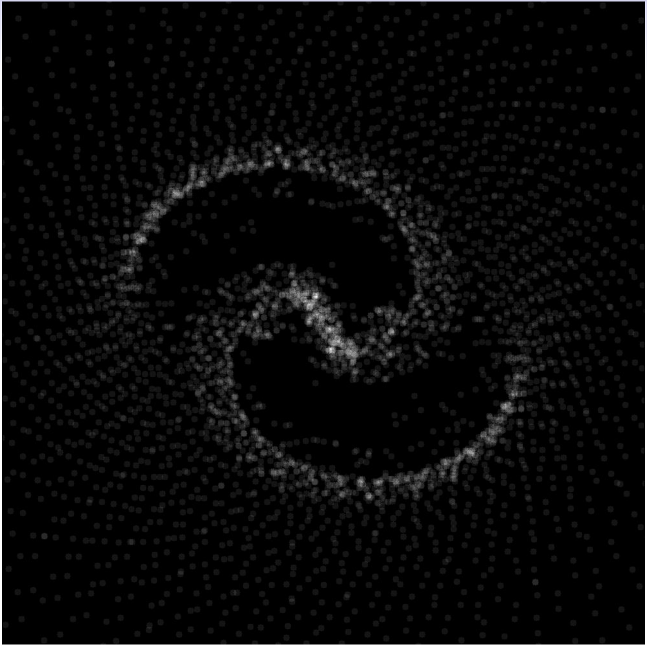


Figure: $t = 22$ million years for Spiral Galaxy Simulation #1.

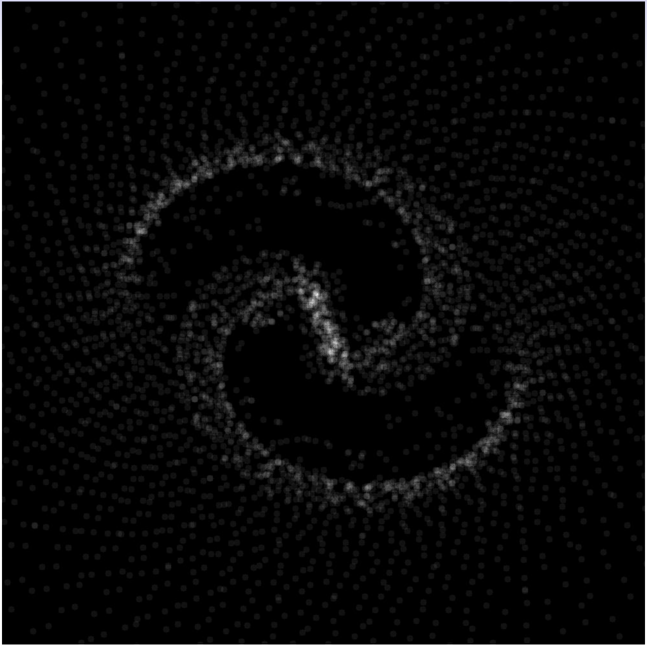


Figure: $t = 23$ million years for Spiral Galaxy Simulation $\#1$.

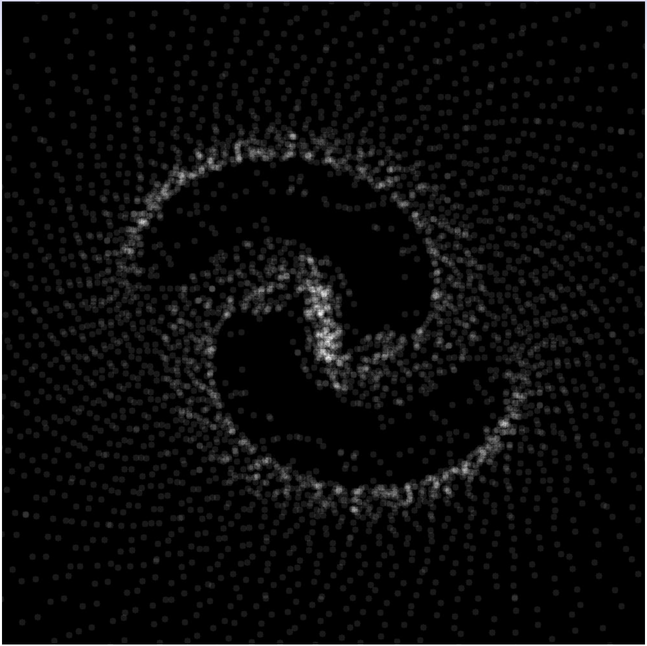


Figure: $t = 24$ million years for Spiral Galaxy Simulation #1.

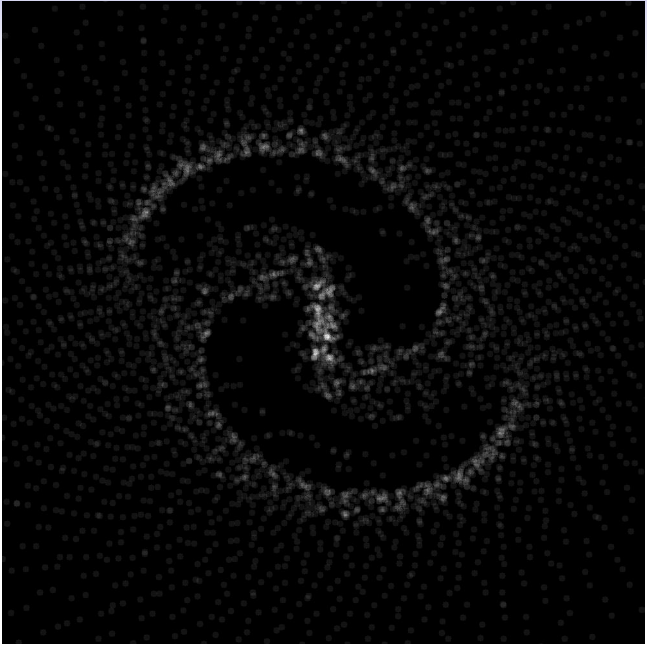
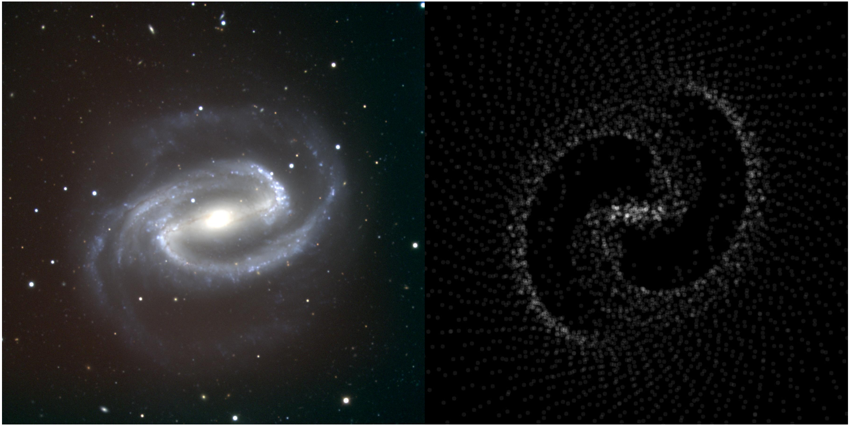


Figure: $t = 25$ million years for Spiral Galaxy Simulation #1.

Spiral Galaxy Simulation #1



NGC1300 on the left, simulation on the right.

Spiral Galaxy Simulation #3

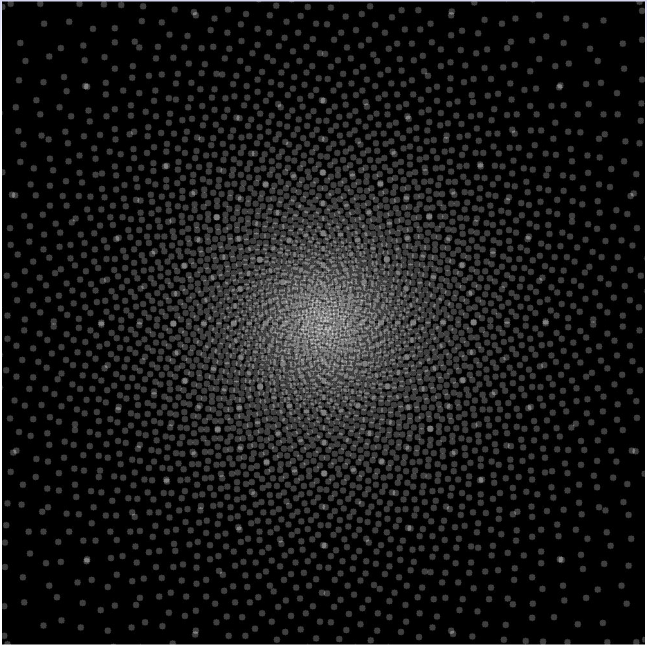


Figure: $t = 0$ million years for Spiral Galaxy Simulation #3.

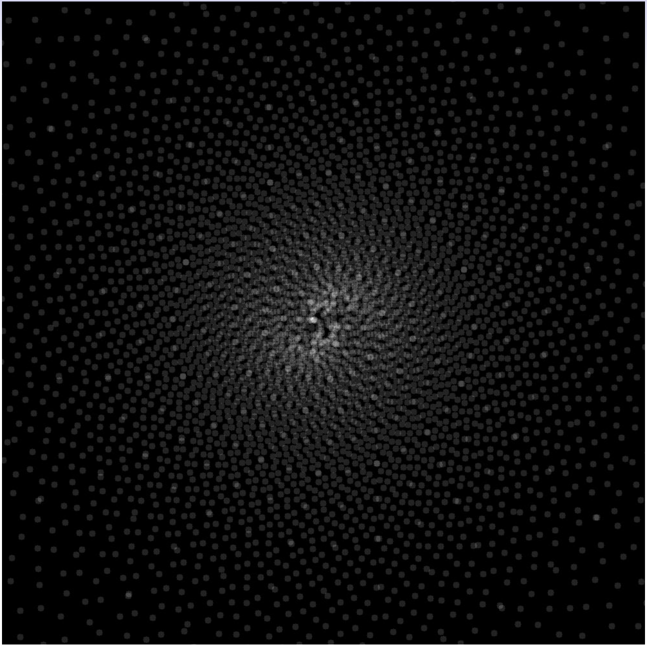


Figure: $t = 5$ million years for Spiral Galaxy Simulation #3.

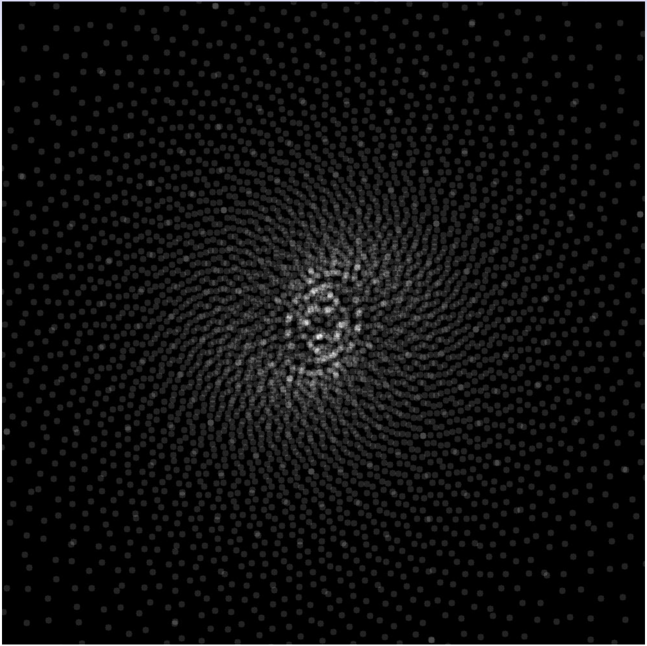


Figure: $t = 10$ million years for Spiral Galaxy Simulation #3.

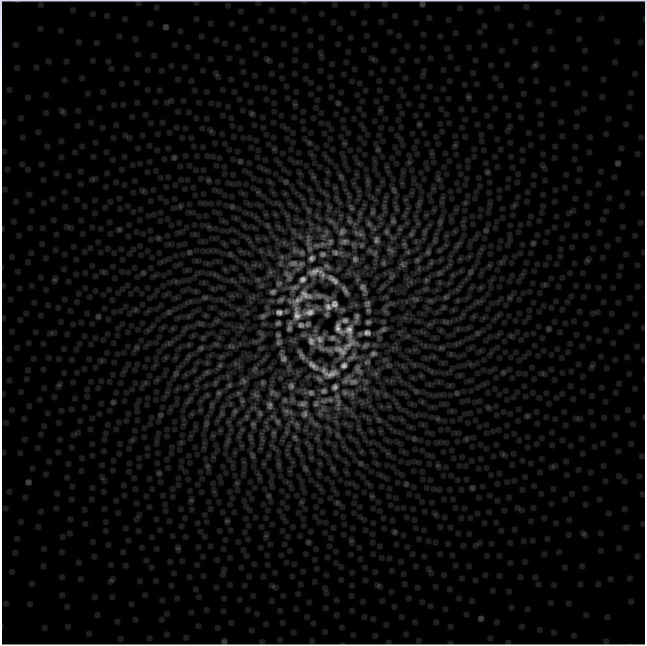


Figure: $t = 15$ million years for Spiral Galaxy Simulation #3.

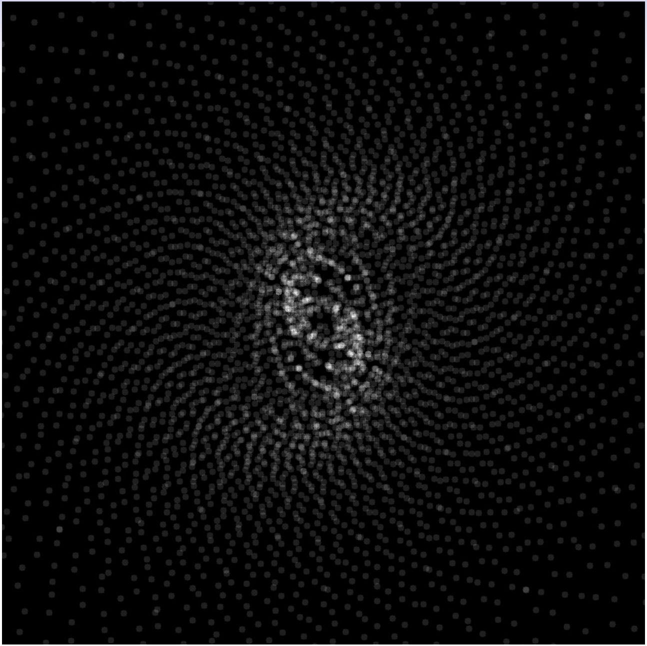


Figure: $t = 20$ million years for Spiral Galaxy Simulation #3.

Figure: $t = 25$ million years for Spiral Galaxy Simulation #3.

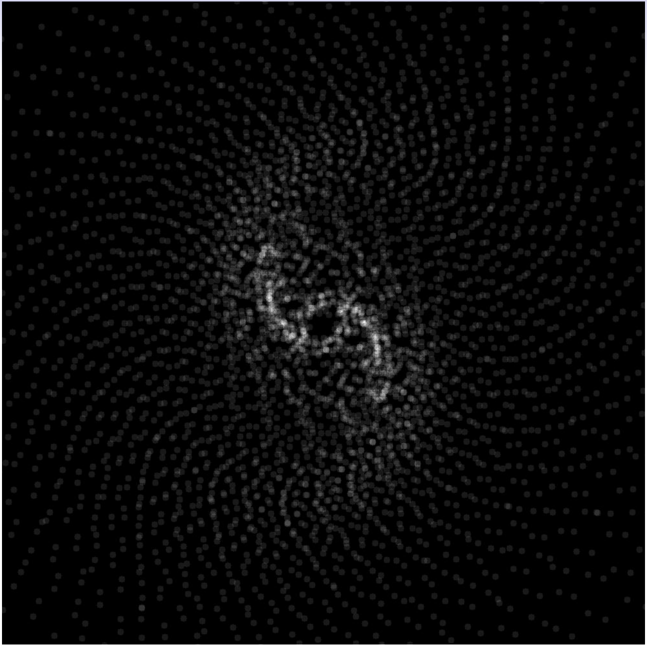


Figure: $t = 30$ million years for Spiral Galaxy Simulation $\#3$.

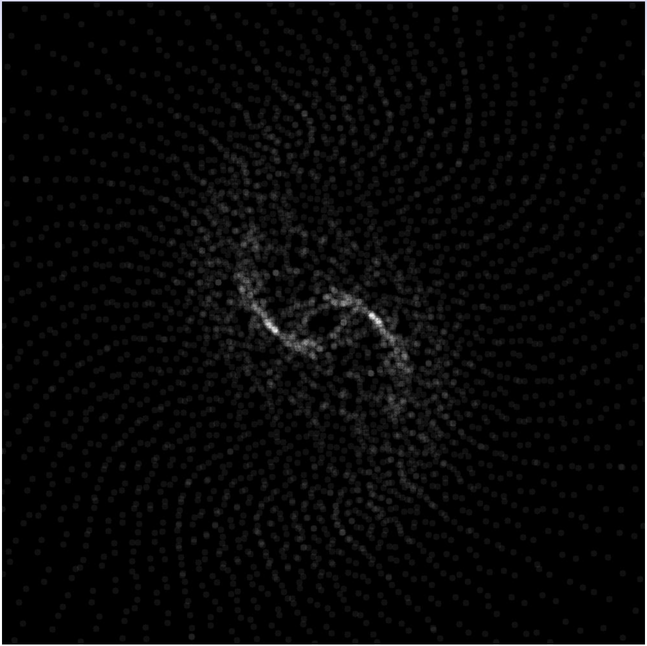


Figure: $t = 35$ million years for Spiral Galaxy Simulation #3.

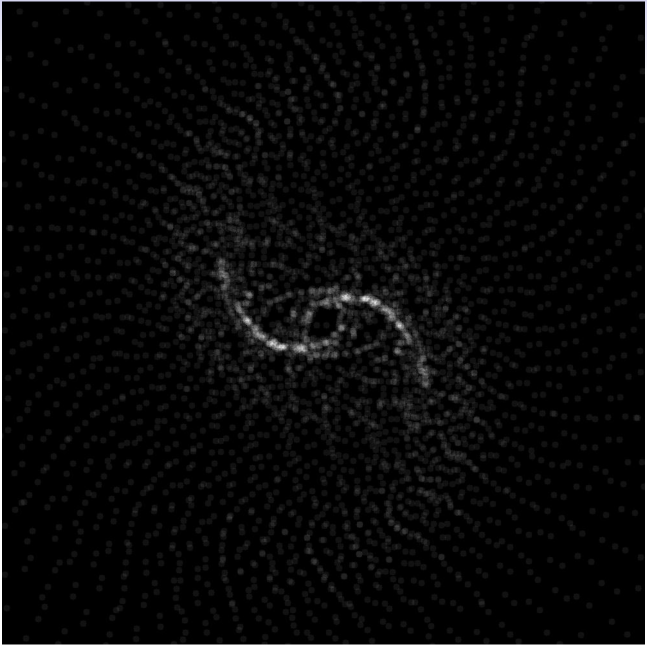


Figure: $t = 40$ million years for Spiral Galaxy Simulation #3.

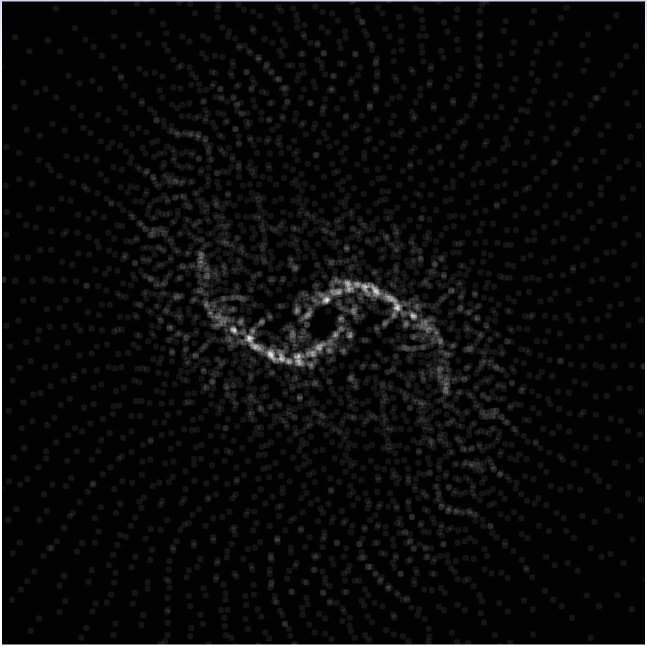


Figure: $t = 45$ million years for Spiral Galaxy Simulation #3.

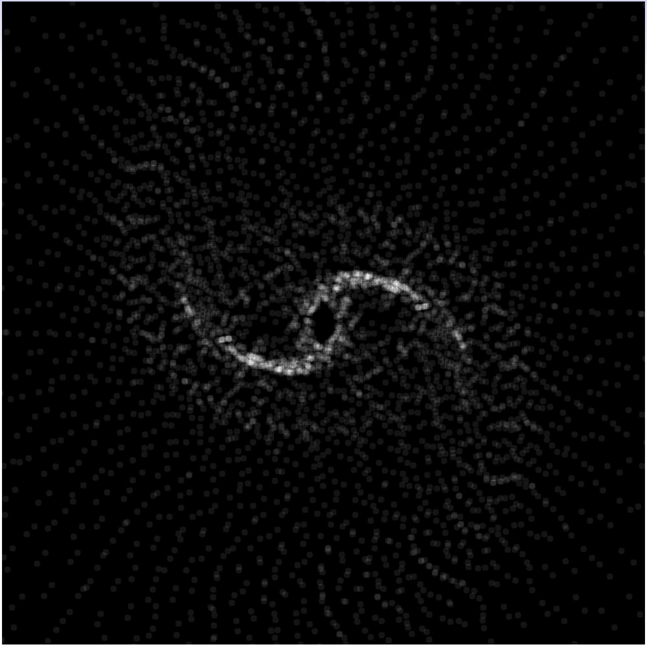


Figure: $t = 50$ million years for Spiral Galaxy Simulation $\#3$.

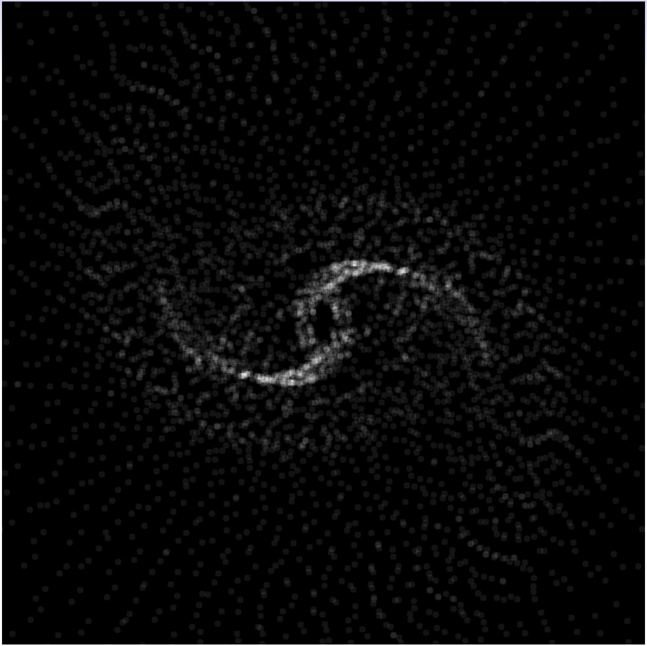


Figure: $t = 55$ million years for Spiral Galaxy Simulation $\#3$.

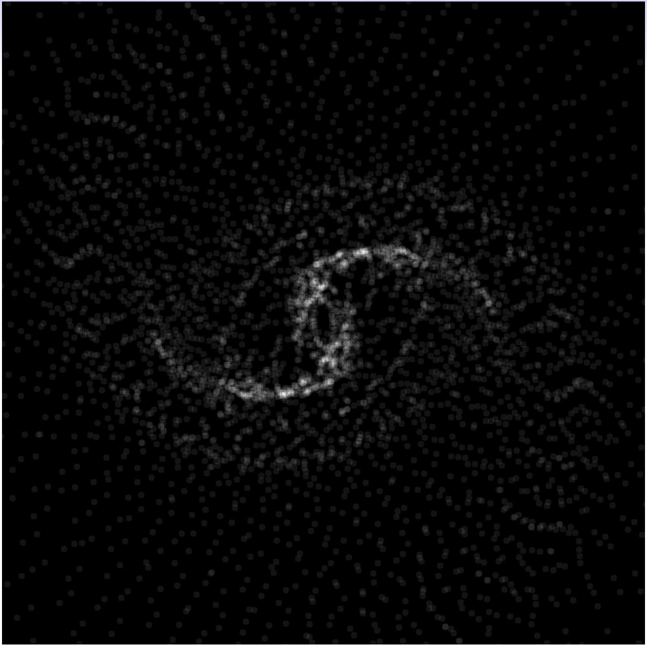


Figure: $t = 60$ million years for Spiral Galaxy Simulation #3.

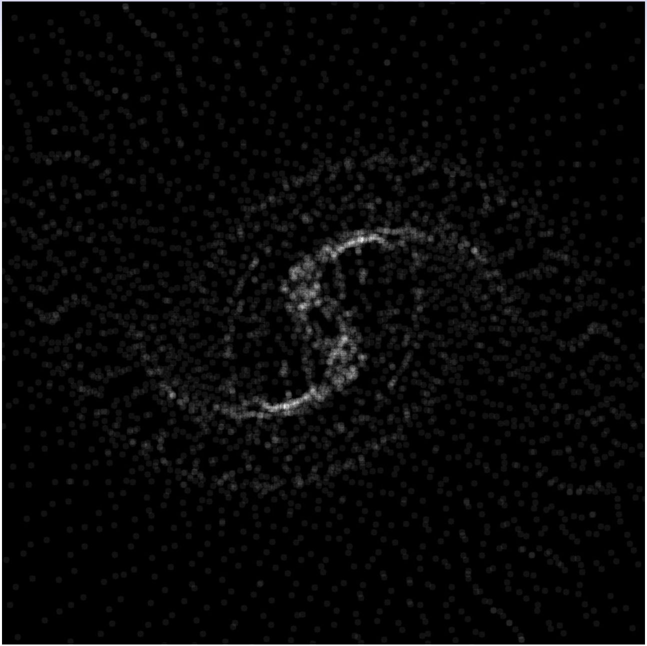


Figure: $t = 65$ million years for Spiral Galaxy Simulation #3.

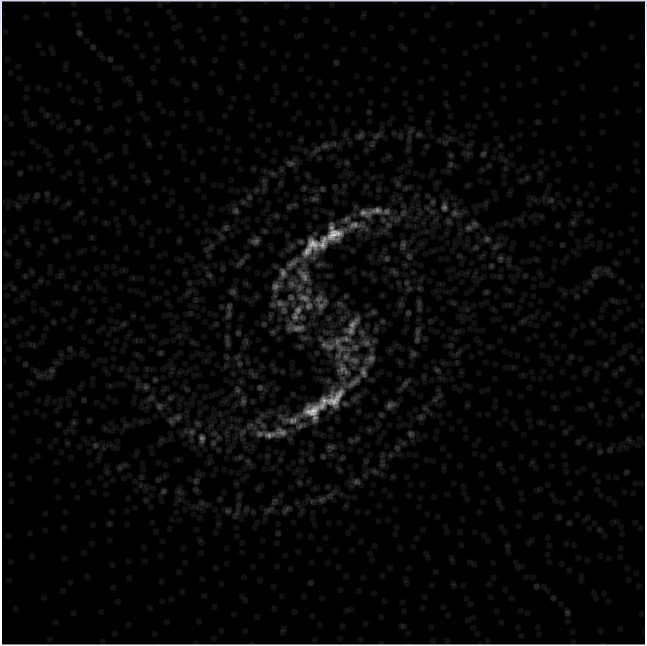


Figure: $t = 70$ million years for Spiral Galaxy Simulation $\#3$.

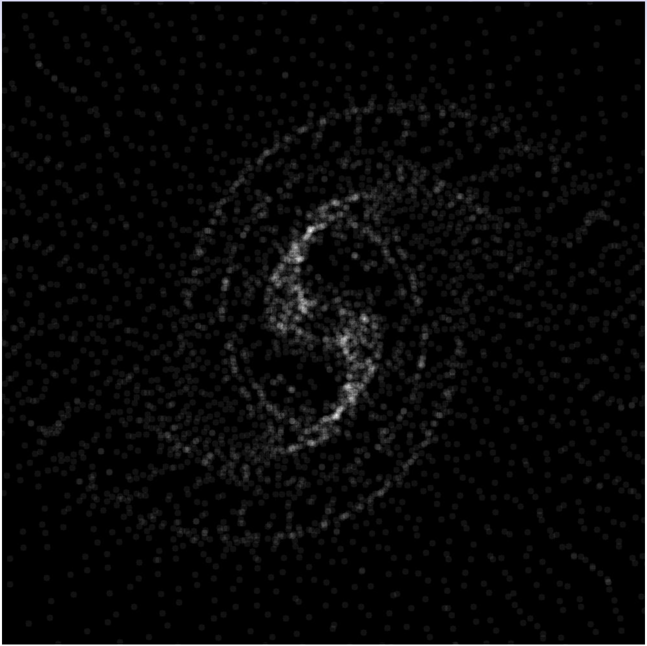


Figure: $t = 75$ million years for Spiral Galaxy Simulation #3.

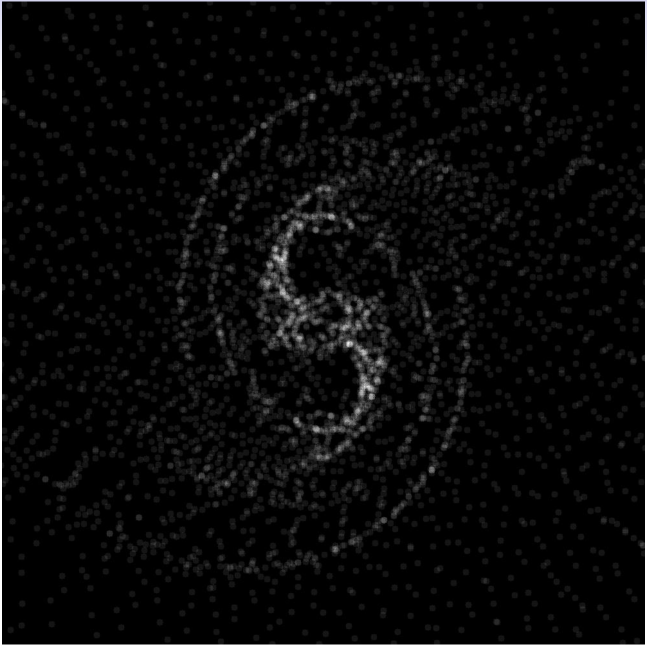


Figure: $t = 80$ million years for Spiral Galaxy Simulation $\#3$.

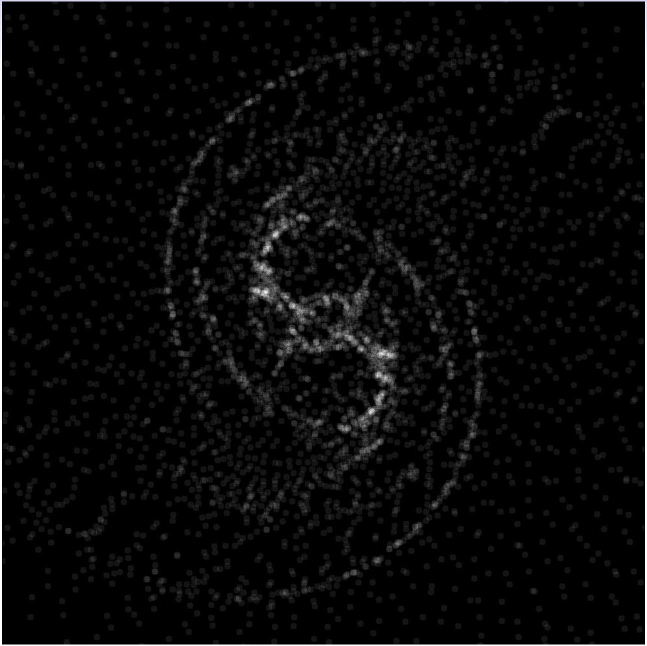


Figure: $t = 85$ million years for Spiral Galaxy Simulation #3.

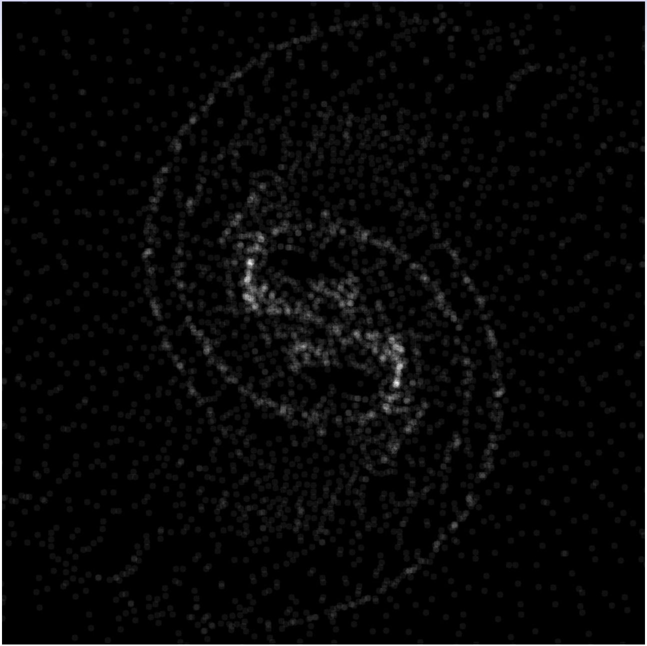


Figure: $t = 90$ million years for Spiral Galaxy Simulation #3.

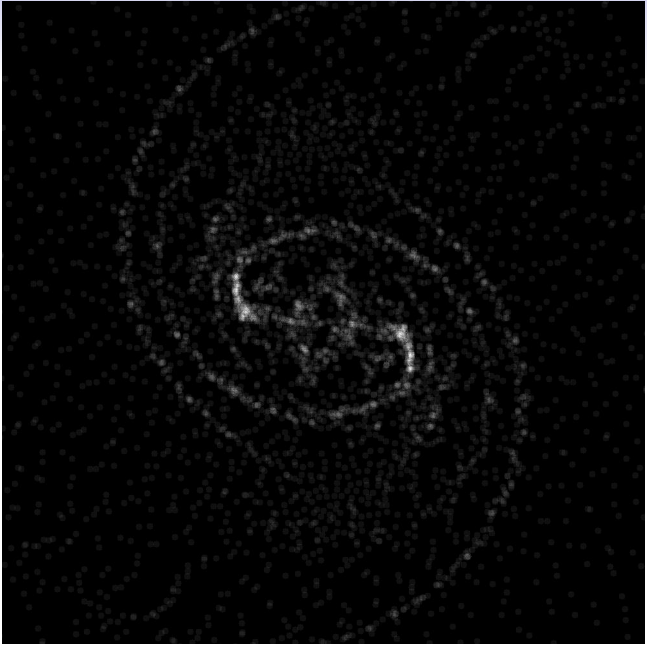
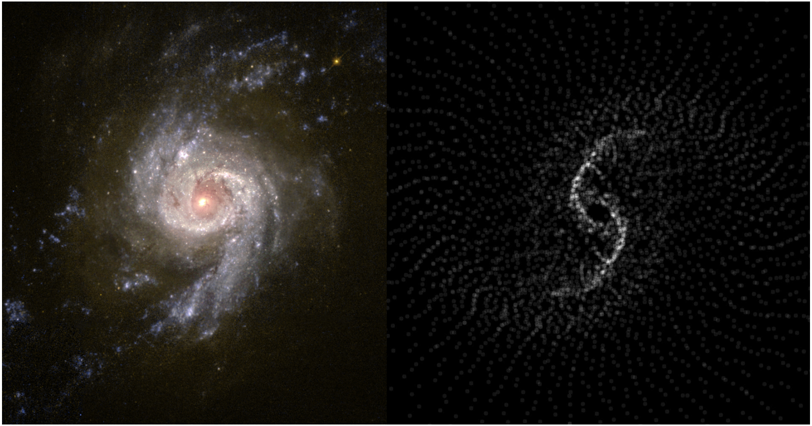


Figure: $t = 95$ million years for Spiral Galaxy Simulation $\#3$.

Spiral Galaxy Simulation #3



NGC3310 on the left, simulation on the right.

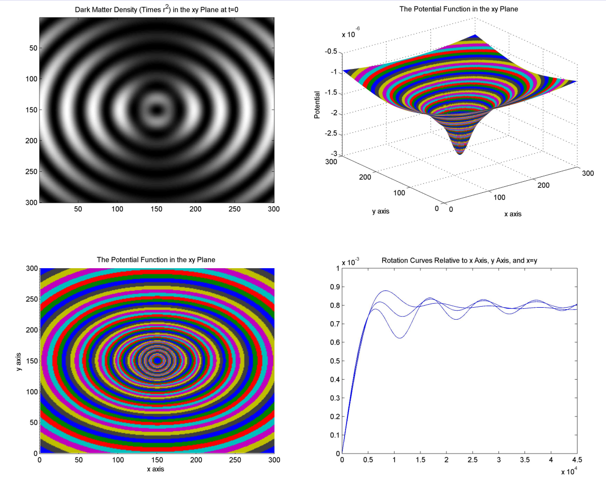
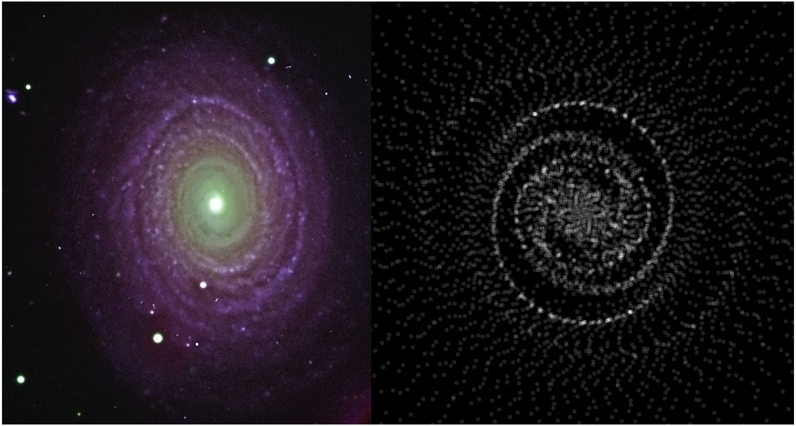


Figure: Spiral Galaxy Simulation #4: The dark matter density times r^2 in the xy plane (top left), the potential function in the xy plane (top right), the level sets of the potential function in the xy plane (bottom left), and the rotation curve (bottom right), all to a radius of $45,000$ light years.

Spiral Galaxy Simulation #4



NGC488 on the left, simulation on the right.

Elliptical Galaxies

Figure: Elliptical galaxies contain ellipsoidal shaped collections of stars in mostly radial orbits. Two examples are M87 (left) and NGC1132 (right).

Ripples in the Brightness Profiles of Elliptical Galaxies

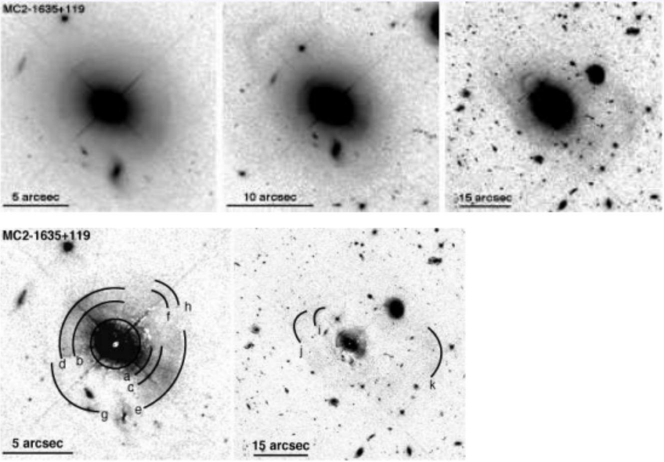
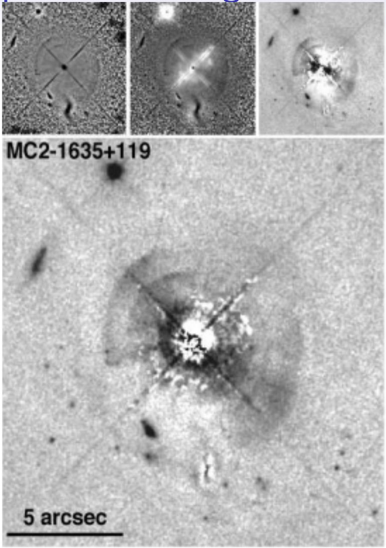


Figure: From "Spectacular Shells in the Host Galaxy of the QSO MC2 1635+119" by Canalizo, Bennert, Jungwiert, Stockton, Schweizer, Lacy, Peng (2007), Astrophysics Journal and on the arXiv.

Ripples in the Brightness Profiles of Elliptical Galaxies



Do these ripples come from a degree 1 spherical harmonic component to the dark matter scalar field solution to the Klein-Gordon equation?

Ripples in the Brightness Profiles of Elliptical Galaxies

Between 10% and 20% of all elliptical galaxies are found to contain sharp steps in their luminosity profiles like those just shown. These features are called ripples or shells and have been observed since 1980.

In *Galactic Astronomy* (1999), Binney and Merrifield write:

"...the existence of ripples directly challenges the classical picture of ellipticals. ...simulations have successfully reproduced the interleaved property of ripples ... Despite these successes significant uncertainties still surround the ripple phenomenon because the available simulations have important limitations, and it is not clear how probable their initial conditions are."

Ripples in the Brightness Profiles of Elliptical Galaxies

Figure: NGC474 on the left, NGC4382 on the right have unusually prominent concentric shells, also known as ripples, in their images.

Ripples in the Brightness Profiles of Elliptical Galaxies

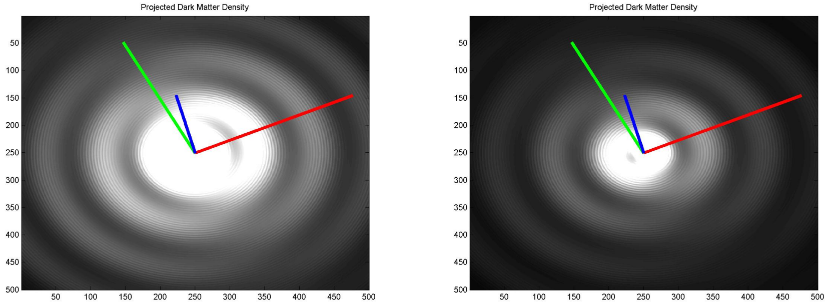


Figure: Projected wave dark matter density. Wave dark matter may offer another possibility to explain ripples in elliptical galaxies, but more study is required.

Ripples in the Brightness Profiles of Elliptical Galaxies

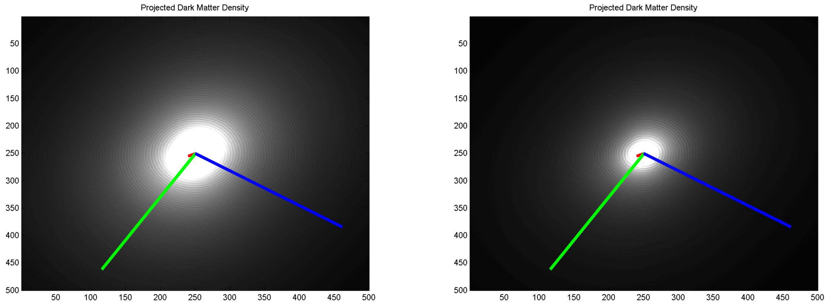
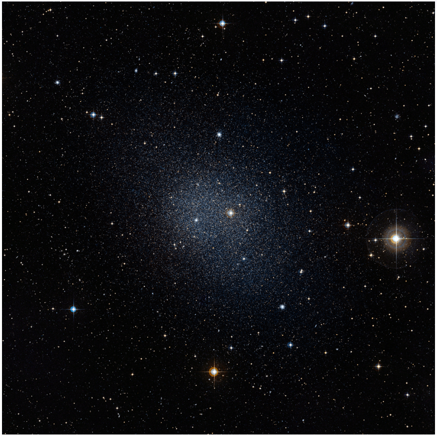


Figure: Same wave dark matter density, but viewed from a different angle where the ripples are not visible.

Dwarf Spheroidal Galaxies



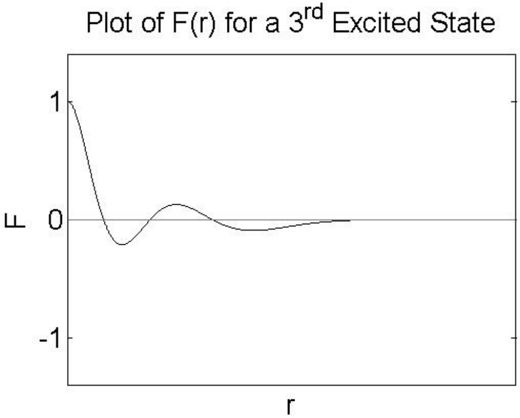
Fornax Dwarf Spheroidal Galaxy. Photo Credit: ESO/Digital Sky Survey 2

Approximately spherically symmetric, and
almost entirely dark matter, over 99% in some cases.

Working Value of Υ (joint with Alan Parry)

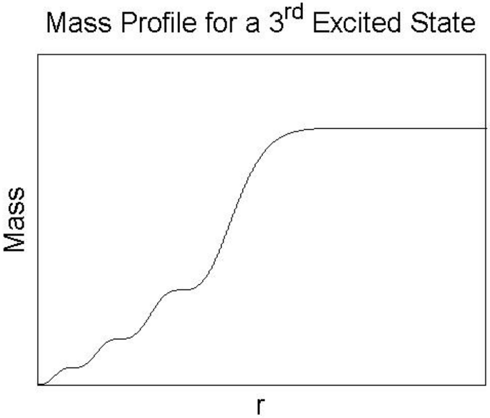
- For $\Upsilon = 50$ yr^{-1}, there exists at least one n^{th} excited state dark matter mass profile for some $n \leqslant 3$ which is qualitatively similar to the Burkert dark matter mass profile found by Salucci et al. for each of the classical dwarf spheroidal galaxies.

3^{rd} Excited State



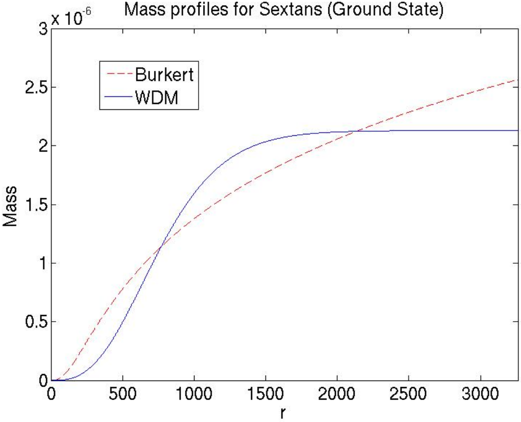
Plot of the scalar field F in a
static spherically symmetric 3^{rd} excited state.

3^{rd} Excited State

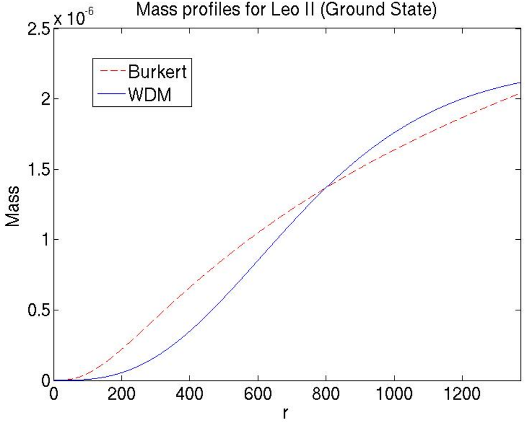


Plot of the mass M in a
static spherically symmetric 3^{rd} excited state.

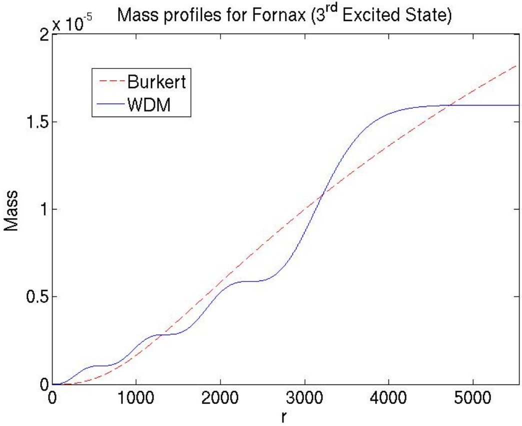
$\Upsilon = 50 \text{ yr}^{-1}$



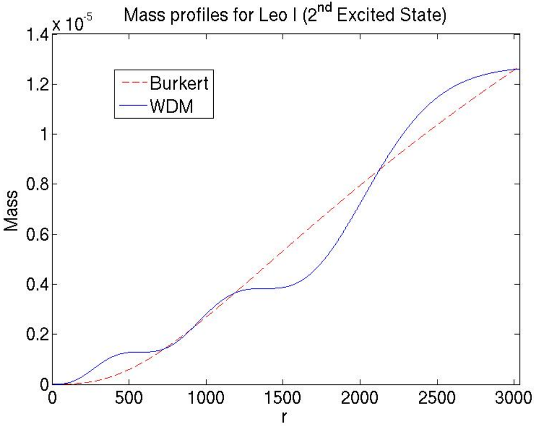
$\Upsilon = 50\ \text{yr}^{-1}$



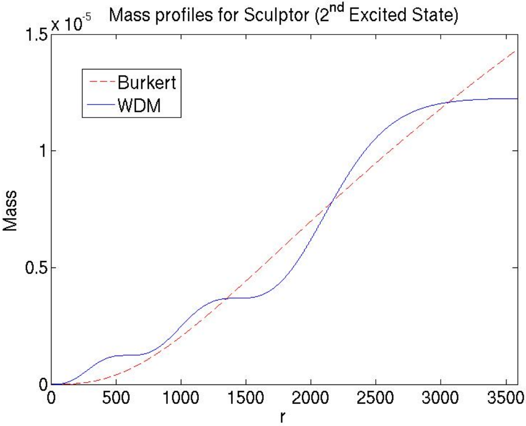
$\Upsilon = 50 \text{ yr}^{-1}$



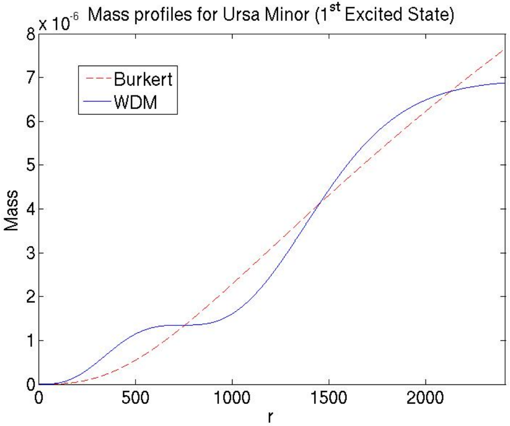
$\Upsilon = 50 \text{ yr}^{-1}$



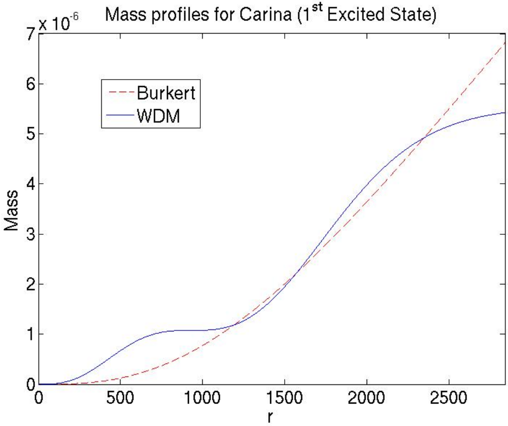
$\Upsilon = 50\ \mathrm{yr}^{-1}$



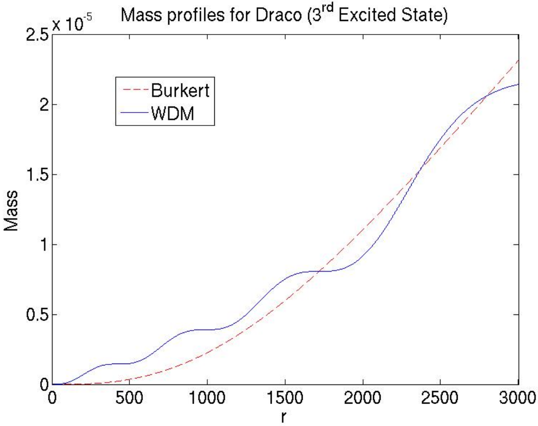
$\Upsilon = 50\ \mathrm{yr}^{-1}$



$\Upsilon = 50\ \text{yr}^{-1}$



$\Upsilon = 50 \text{ yr}^{-1}$



Upper Bounds on Υ

Galaxy \ State	0	1	2	3
Sextans	$\Upsilon < 160$	$\Upsilon < 394$	$\Upsilon < 633$	$\Upsilon < 875$
Leo II	$\Upsilon < 234$	$\Upsilon < 576$	$\Upsilon < 926$	$\Upsilon < 1279$
Fornax	$\Upsilon < 35$	$\Upsilon < 87$	$\Upsilon < 139$	$\Upsilon < 192$
Leo I	$\Upsilon < 57$	$\Upsilon < 141$	$\Upsilon < 226$	$\Upsilon < 312$
Sculptor	$\Upsilon < 49$	$\Upsilon < 121$	$\Upsilon < 194$	$\Upsilon < 268$
Ursa Minor	$\Upsilon < 82$	$\Upsilon < 202$	$\Upsilon < 325$	$\Upsilon < 449$
Carina	$\Upsilon < 84$	$\Upsilon < 207$	$\Upsilon < 333$	$\Upsilon < 459$
Draco	$\Upsilon < 45$	$\Upsilon < 111$	$\Upsilon < 179$	$\Upsilon < 246$

Galaxy \ State	4	5	10	20
Sextans	$\Upsilon < 1116$	$\Upsilon < 1356$	$\Upsilon < 2460$	$\Upsilon < 4789$
Leo II	$\Upsilon < 1632$	$\Upsilon < 1983$	$\Upsilon < 3597$	$\Upsilon < 7003$
Fornax	$\Upsilon < 245$	$\Upsilon < 297$	$\Upsilon < 538$	$\Upsilon < 1048$
Leo I	$\Upsilon < 398$	$\Upsilon < 484$	$\Upsilon < 877$	$\Upsilon < 1706$
Sculptor	$\Upsilon < 342$	$\Upsilon < 416$	$\Upsilon < 754$	$\Upsilon < 1467$
Ursa Minor	$\Upsilon < 572$	$\Upsilon < 695$	$\Upsilon < 1261$	$\Upsilon < 2455$
Carina	$\Upsilon < 586$	$\Upsilon < 712$	$\Upsilon < 1292$	$\Upsilon < 2514$
Draco	$\Upsilon < 314$	$\Upsilon < 382$	$\Upsilon < 692$	$\Upsilon < 1347$

Summary of Dwarf Spheroidal Galaxy Results (joint with Alan Parry)

Conclusion 1 For $\Upsilon = 50$ yr^{-1}, there exists at least one n^{th} excited state dark matter mass profile for some $n \leqslant 3$ which is qualitatively similar to the Burkert dark matter mass profile found by Salucci et al. for each of the classical dwarf spheroidal galaxies.

Conclusion 2 Under a precise criteria to reject values of Υ as untenable, if the dark matter mass in all eight dwarf spheroidal galaxies are correctly modeled by a 20^{th} excited state or less, then $\Upsilon < 1000$ yr^{-1}.

The Baryonic Tully-Fisher Relation (McGaugh, 2000)

For disk galaxies, measure/estimate

- M_b - baryonic mass
- v - rotational velocity (average, maximum, etc.)

The Baryonic Tully-Fisher Relation

$$M_b \propto v^x$$

Survey of literature shows $3 \leqslant x \leqslant 4$.

The Baryonic Tully-Fisher Relation

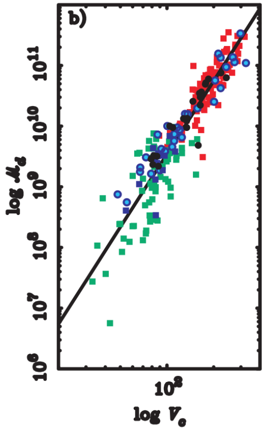


Figure: The Baryonic Tully-Fisher Relation. McGaugh et al. 2000, ApJ, 533, L99

The Einstein-Klein-Gordon Equations

Einstein's equation:

$$G + \Lambda g = 8\pi \left(\frac{df \otimes d\bar{f} + d\bar{f} \otimes df}{\Upsilon^2} - \left(\frac{|df|^2}{\Upsilon^2} + |f|^2 \right) g \right)$$

Klein-Gordon equation:

$$\Box_g f = \Upsilon^2 f$$

The Action of the Universe

Consider the following action:

$$\mathcal{F}(g, f, A) = \int \left[R - 2\Lambda - 16\pi \left(\frac{|df|^2}{\Upsilon^2} + |f|^2 + \frac{1}{4}|dA|^2 \right) \right] dV.$$

- g - Lorentzian metric
- f - complex-valued scalar function (wave dark matter)
- A - one-form (electromagnetic potential)

Requiring g, f, and A to be critical points of this functional leads to the Einstein-Klein-Gordon-Maxwell equations.

This action might describe most of the contents of the universe at most times.

The EKG Equations in Spherical Symmetry

Ansätze

The metric ansatz:

$$g = -e^{2V(r)}\,dt^2 + \left(1 - \frac{2M(r)}{r}\right)^{-1} dr^2 + r^2\,d\theta^2 + r^2 \sin^2\theta\,d\phi^2.$$

The static state ansatz:

$$f(t,r) = F(r)e^{i\omega t}$$

Definition

$$\Phi(r) = 1 - \frac{2M(r)}{r}$$

The EKG Static State ODEs
The ODEs

$$M_r = 4\pi r^2 \cdot \frac{1}{\Upsilon^2}\left[\left(\Upsilon^2 + \omega^2 e^{-2V}\right)F^2 + \Phi F_r^2\right]$$

$$\Phi V_r = \frac{M}{r^2} - 4\pi r \cdot \frac{1}{\Upsilon^2}\left[\left(\Upsilon^2 - \omega^2 e^{-2V}\right)F^2 - \Phi F_r^2\right]$$

$$F_{rr} + \frac{2}{r}F_r + V_r F_r + \frac{1}{2}\frac{\Phi_r}{\Phi}F_r = \Phi^{-1}\left(\Upsilon^2 - \omega^2 e^{-2V}\right)F$$

Initial Conditions

$$M(0) = 0 \qquad F(0) = F_0 > 0$$

$$V(0) = V_0 \qquad F_r(0) = 0$$

The Poisson-Schrödinger ODEs

The ODEs

$$M_r = 4\pi r^2 \cdot 2F^2$$

$$V_r = \frac{M}{r^2}$$

$$\frac{1}{2\Upsilon}\left(F_{rr} + \frac{2}{r}F_r\right) = (\Upsilon - \omega + \Upsilon V)F$$

Initial Conditions

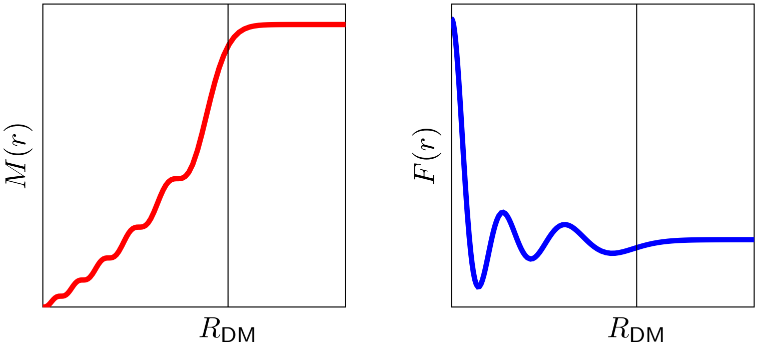
$$M(0) = 0 \qquad F(0) = F_0 > 0$$

$$V(0) = V_0 \qquad F_r(0) = 0$$

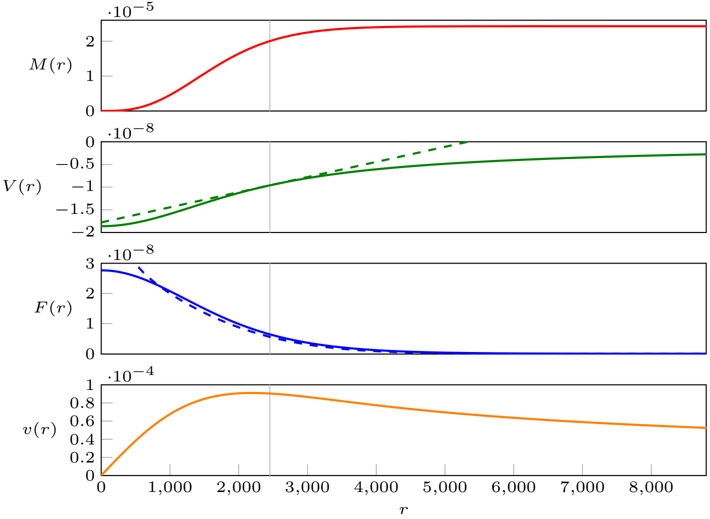
Definition of R_{DM}

Definition

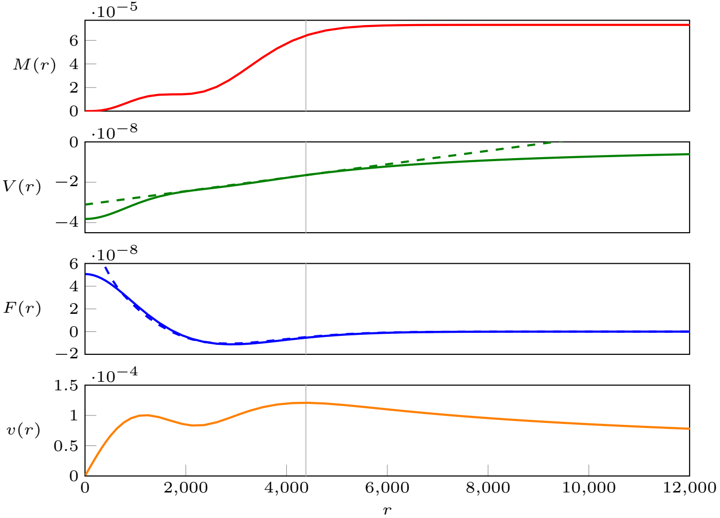
We define R_{DM} to be the radius at which the function F switches from oscillatory to exponential behavior.



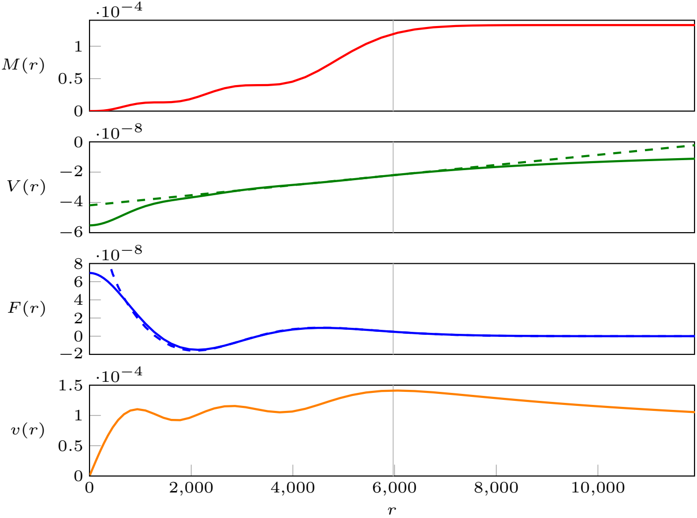
A Ground State ($n = 0$)



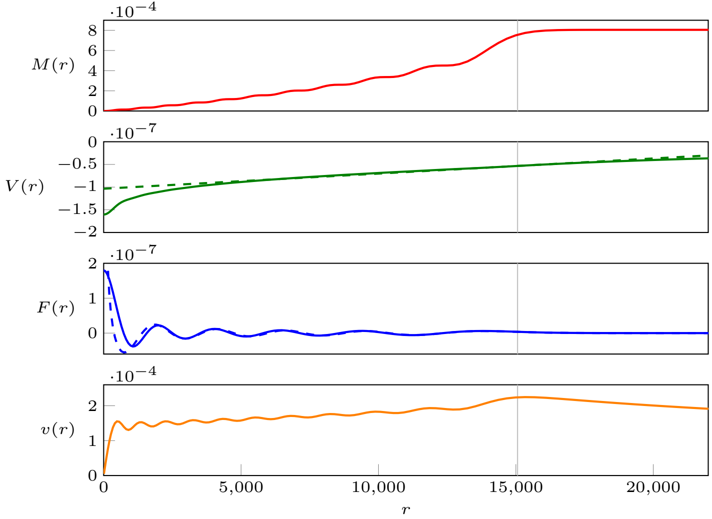
A First Excited State ($n = 1$)



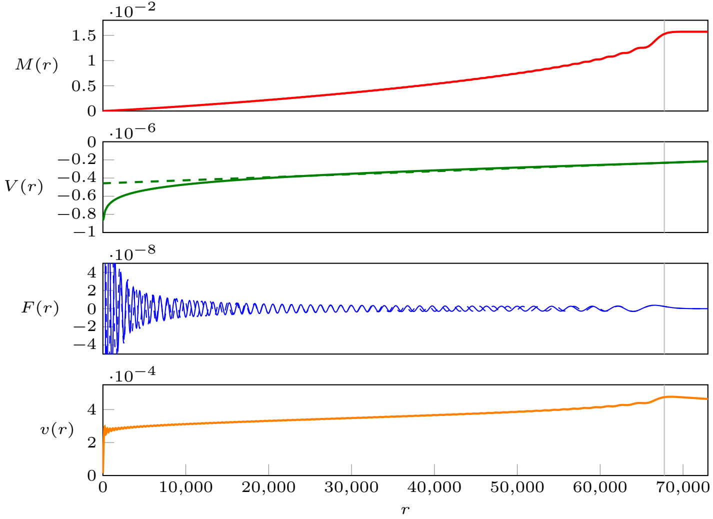
A Second Excited State ($n = 2$)



A Tenth Excited State ($n = 10$)



A Hundredth Excited State ($n = 100$)



Scalings of the Static States

Theorem

Suppose $(\omega; M, V, F)$ *solves the Poisson-Schrödinger system for a particular value of* Υ*. Let* $\alpha, \beta > 0$*. Then* $(\bar{\omega}; \bar{M}, \bar{V}, \bar{F})$ *defined by*

$$\begin{aligned}
\bar{r} &= \alpha^{-1}\beta^{-1} r \\
\bar{M} &= \alpha\beta^{-3} M \\
\bar{V} &= \alpha^2\beta^{-2} V \\
\bar{F} &= \alpha^2 F \\
\bar{\Upsilon} &= \beta^2 \Upsilon \\
(\bar{\Upsilon} - \bar{\omega}) &= \alpha^2 (\Upsilon - \omega)
\end{aligned}$$

is also a solution of the Poisson-Schrödinger system with Υ *replaced by* $\bar{\Upsilon}$*.*

Scalings of the Static States

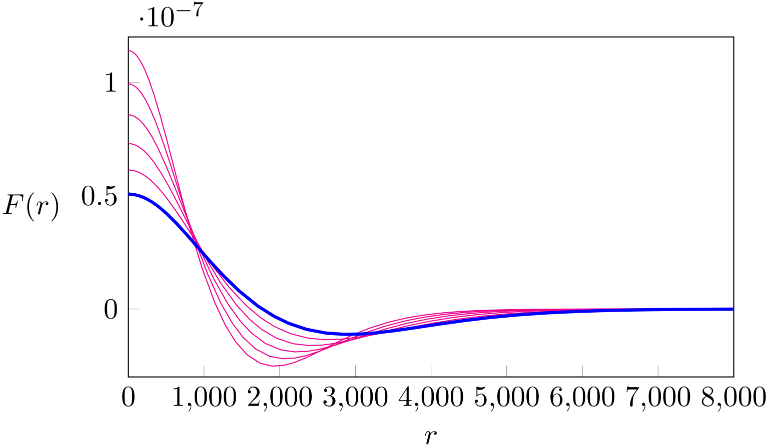


Figure: Five scalings of an $n = 1$ excited state (shown in blue).

Parametrizations of the Static States

Fact

The static states form a 3-parameter family: 2 continuous parameters, 1 discrete parameter.

Examples:

- Υ, n, ω
- $\Upsilon, n, R_{\mathsf{DM}}$
- $\Upsilon, M(R_{\mathsf{DM}}), R_{\mathsf{DM}}$
- $\Upsilon, n,$???

Fixing a Scaling

Choosing something to replace ??? "fixes a scaling" and gives a unique sequence of static states $n = 0, 1, 2, \ldots$.

The Main Ideas

Sin's Idea (1994)

Dark matter in galaxies is approximately spherically symmetric. Each halo corresponds to a static state of some order n.

Our Idea (joint with Andrew Goetz)

The BTFR exists because of the scaling properties of the static states.

Question

Are there any natural scaling conditions that give a Tully-Fisher-like relation for the static states?

The Fundamental Question

Question

Are there are any scaling conditions for which the sequence of static states $n = 0, 1, 2, \ldots$ obeys a Tully-Fisher-like relation

$$\frac{M(R_{\mathsf{DM}})}{v(R_{\mathsf{DM}})^x} = \text{constant}$$

for some $3 \leqslant x \leqslant 4$?

Answer

BC1: Fixing a length scale at R_{DM} gives slope $x = 4$.
BC2: Fixing $|F(R_{\mathsf{DM}})|$ gives slope $x = 3.4$.

BC1 and BC2

Boundary Condition 1 (joint with Andrew Goetz)

What does it mean to fix a length scale? Example: Fix the halflength of F. Another example: Fix the distance between the last two nodes.

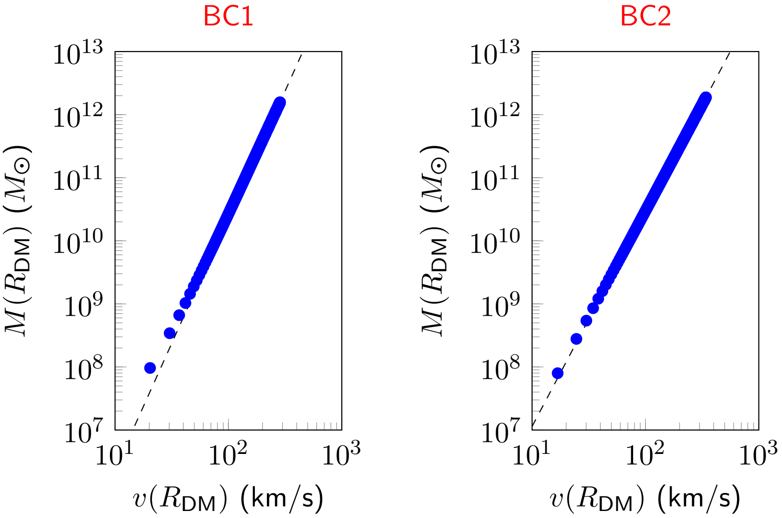
Boundary Condition 2 (entirely the idea of Andrew Goetz)

Fixing $|F(R_{\text{DM}})|$ is equivalent to fixing the density of the dark matter at the outer edge.

I favor BC2:

Mathematically, it's much like a Dirichlet boundary condition. Physically, it's the idea that solutions match up with an average background density of dark matter. If one accepts this physical motivation, then static wave dark matter solutions **predict** a Baryonic Tully-Fisher relation with exponent 3.4, consistent with observations.

Numerical Evidence for BC1 and BC2



The Baryonic Tully-Fisher Relation

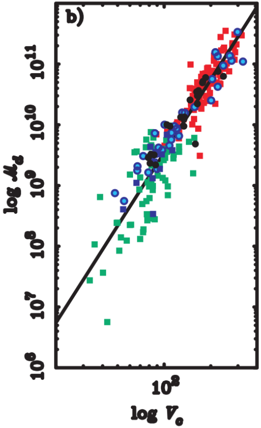
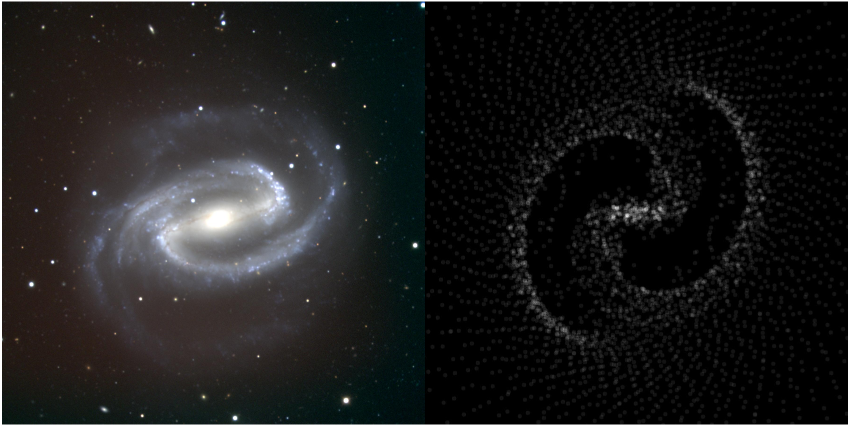


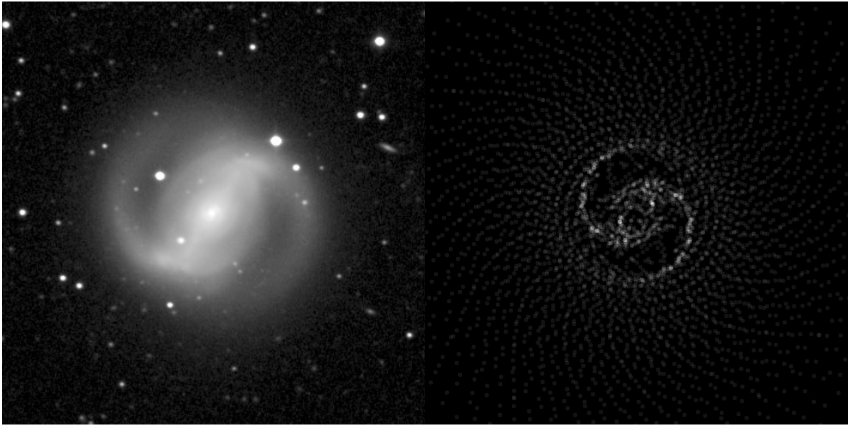
Figure: The Baryonic Tully-Fisher Relation. McGaugh et al. 2000, ApJ, 533, L99

Spiral Galaxy Simulation #1



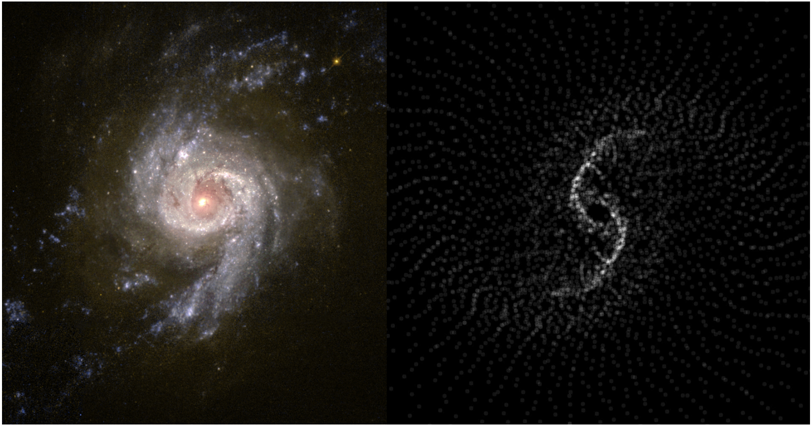
NGC1300 on the left, simulation on the right.

Spiral Galaxy Simulation #2



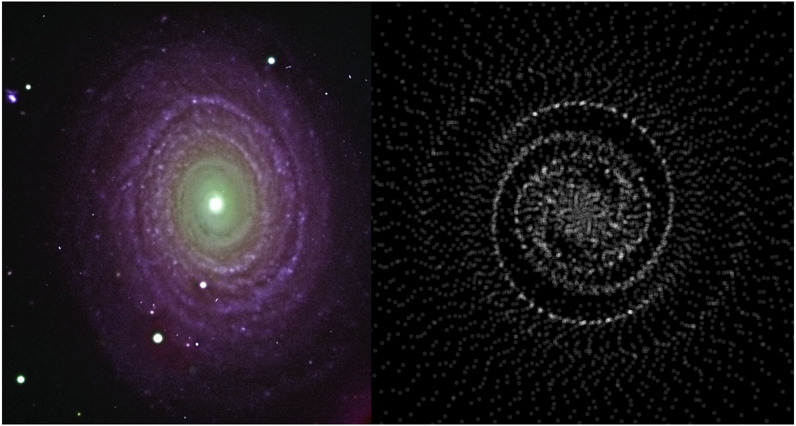
NGC4314 on the left, simulation on the right.

Spiral Galaxy Simulation #3



NGC3310 on the left, simulation on the right.

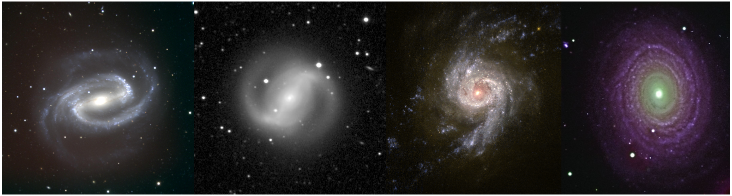
Spiral Galaxy Simulation #4



NGC488 on the left, simulation on the right.

Arguments in Favor of Wave Dark Matter

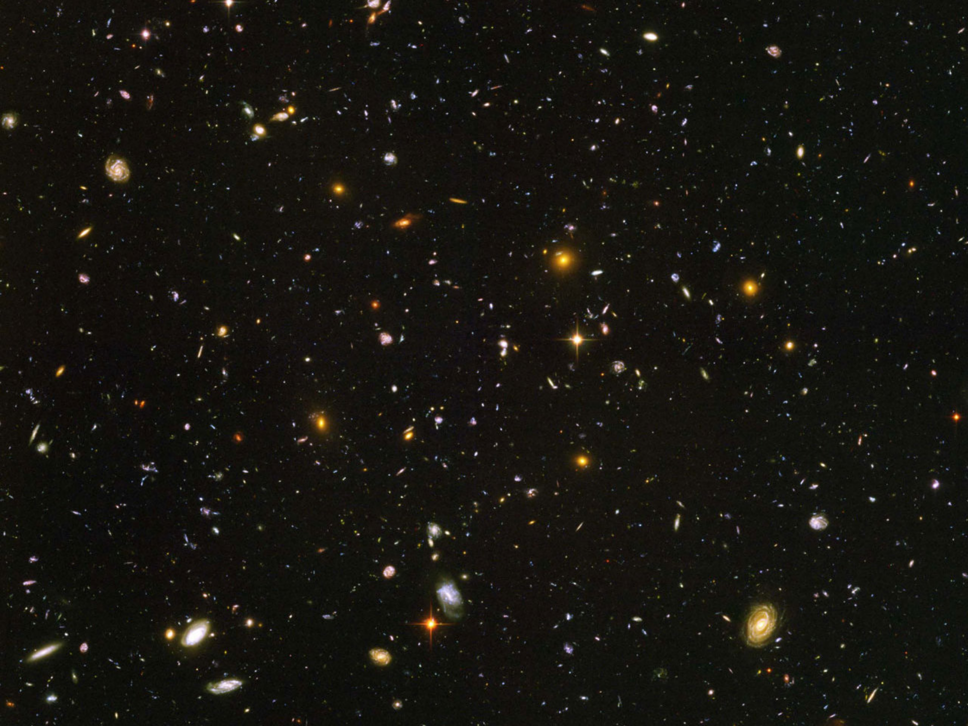
1. Geometrically natural
2. Spiral patterns in galaxies



3. The Tully-Fisher relation with exponent 3.4 (Andrew Goetz)

as well as

- The lower bound on the mass of galaxies
- The coldness of dark matter
- The interleaved shells in some elliptical galaxies.
- Bounded dark matter densities in the cores of galaxies



Math Conjecture

It is known that the excited states are unstable by themselves but that the presence of regular matter can stabilize them.

Stability Conjecture

In the low-field, nonrelativistic limit, for each choice of total regular mass M_b and regular matter radius R_b, there exists an integer $N \geqslant 0$ such that static, spherically symmetric solutions to the EKG system satisfying the boundary condition BC1 or BC2 with $n \leqslant N$ are stable and those with $n > N$ are unstable.

Physics Conjecture

If the math conjecture is true, we make the following physics conjecture:

Dark Matter Saturation Conjecture

The dark matter and total matter distributions of most galaxies which are approximately static and spherically symmetric are approximately described by static, spherically symmetric solutions to the EKG system satisfying the boundary condition BC1 or BC2 with $n = N$.