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Gravitational Waves

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Introduction

First proposed in November of 1915, "Einstein's general theory of relativity is the expression of our understanding of gravity" (Stannard, 2008). General relativity, since its conception, has proven valid in every experimentation and observation. From time dilation of satellites to the bending of light to the orbit of Mercury, even the discovery of black holes has resulted from impressive mathematical manipulation of Einstein's equations describing gravity and spacetime. A hundred years later, on September 14th, 2015, the final piece of the theory of general relativity was confirmed: the discovery of gravitational waves (Cervantes-Cota, 2016). Perhaps the most important scientific discovery of the century so far, the discovery of gravitational waves was not simple. Gravitational waves were challenged by many scientists, including Einstein, and causes effects so weak that some concluded that gravitational waves could never be detected. But first, to understand what a gravitational wave is, we must ask: What is the theory of general relativity?

General Relativity

In Newtonian physics, gravity is described as a force between two massive bodies, causing both objects to accelerate towards each other. The force of gravity is defined with the equation:

$$F_g = \frac{Gm_1m_2}{r^2}$$

where Fg is the force of gravity in Newton meters, G is Newton's gravitational constant, m1 and m2 are the masses of the two objects in kilograms, and r is the distance between the two objects in meters (Kruse, 2016).

In general relativity, gravity is not so simple. Gravity is not really the force between two objects, but is a result of the curvature of spacetime. By the equivalence principle of general relativity, there is no difference between gravity and acceleration (Kruse, 2016). If someone is free falling on Earth, they feel the same acceleration as if they were accelerating at 9.8 m/s^2 in space, so gravity is best described as an accelerating inertial reference frame. As part of general relativity, Einstein's equations can be summed up as:

$$\mathfrak{G} = 8\pi G\mathfrak{T}/c^4,$$

(Ismailov, 2016) where $\mathfrak{G}$ is a tensor, referred to as the "Einstein tensor" to "determine changes in the shape of space time" (Ismailov, 2016), G is Newton's gravitational constant, c is the speed of light in a vacuum, and τ is the energy momentum tensor which describes the "mass density" (Ismailov, 2016) in a gravitational field. A tensor is basically a function or object that modifies information, like a dot product, cross product, or linear map (Ismailov, 2016). This equation essentially describes how spacetime is curved by the presence of mass and energy and how this warped spacetime affects particles.

Gravitational Waves

Maxwell predicted the existence of electromagnetic waves that are caused by an acceleration of an electrical charge much before Einstein published his theory of general relativity (Stannard, 2008). These electromagnetic waves are more commonly referred to as light, ranging from radio waves to visible light to gamma rays. In a similar fashion, Einstein predicted the existence of gravitational waves caused by the acceleration of massive bodies (Stannard, 2008). "Gravitational waves can be envisaged as ripples passing through the spacetime fabric" (Stannard, 2008). In general, massive bodies warp spacetime, like a bowling ball on a bed, and gravitational waves cause spacetime to vibrate, like shaking the bedsheet up and down. Like electromagnetic waves, gravitational waves travel at the speed of light (Stannard, 2008).

Unlike electromagnetic waves, however, gravitational waves are extremely difficult to detect. For example, electromagnetic waves are created with very high energies from charged particles in circular accelerators. When accelerated to relativistic speeds, electrons actually lose so much energy in circular accelerators that linear accelerators are much more efficient for collisions. Instead, to "whirl a lump of steel weighing several tons at rotational speeds" close to tearing the steel apart would only emit 10^{-30} watts of gravitational energy (Stannard, 2008), so detecting gravitational waves in this method is highly implausible. This shows that although all accelerating matter can emit gravitational waves, only supermassive bodies will emit a significant amount of gravitational waves. The most significant emitters of gravitational waves would thus be stars or black holes orbiting each other as can be seen in Figure 1., or a supernova explosion.

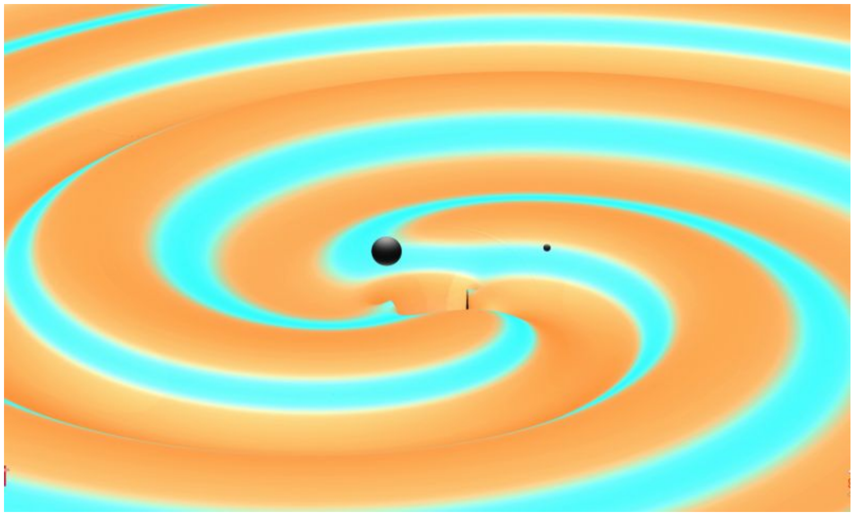

Figure 1. On December 26th, 2015, the gravitational waves resulting from these two black holes merging was detected at both LIGO sites. The orange and blue areas represent peaks and troughs of gravitational waves emitted as the black holes orbit one another (Gravitational, 2016).

So how was detection achieved? We must first go back to the onset of gravitational waves to answer this question.

History of Gravitational Waves

The idea of gravitational waves was first postulated on July 5th, 1905 by Henri Poincare as the method by which the force of gravity was transmitted (Cervantes-Cota, 2016). While the description of Poincare's gravitational waves was incorrect, this idea may have led to Einstein realizing that perhaps gravitational waves are created in his theory of general relativity. While as shown above as a single equation, Einstein's theory of general relativity involves many complex equations, as tensors are complicated objects containing many components, so Einstein's equations cannot have a general solution (Cervantes-Cota, 2016). Instead, many specific solutions involving approximations have been discovered. Solutions to Einstein's equations resulted in the discovery of black holes and other important spacetime curvatures. Currently, computers are being used with advanced techniques called numerical relativity that attempt to find more solutions that involve dense calculations (Cervantes-Cota, 2016).

After publishing the theory of general relativity, one of Einstein's main focuses was to find a solution to his equations to make his equations look like Maxwell's equations for electromagnetic waves. Einstein's major issue in this endeavor was that he was using a coordinate system that made calculations rather difficult, but once Einstein changed to a different coordinate system, Einstein was able to find 3 different types of gravitational waves (Cervantes-Cota, 2016). In 1916, Einstein published his findings, but his work was heavily criticised by scientists, and also heavily doubted by himself. The criticism resulted from the fact that electromagnetic waves are an oscillation between positive and negative charge, but negative mass doesn't exit so oscillating waves of mass made no logical sense. Finally, in 1922, Arthur Eddington, the same mathematician who proved general relativity by observing the curvature of light by the sun during a solar eclipse, showed that 2 of the waves proposed by Einstein were not actually waves (Cervantes-Cota, 2016). These two waves were able to travel at any speed, including faster than light speeds, which was not possible and thus these results just appeared to be waves because of the coordinate system used for Einstein's calculations. The third wave moved at the speed of light and was still determined to be mathematically sound (Cervantes-Cota, 2016).

Despite proposing gravitational waves, Einstein still seriously doubted their existence and, in 1936 with the help of a Soviet scientist named Rosen, published a paper proving that gravitational waves don't exist (Cervantes-Cota, 2016). To Einstein's embarrassment, as his paper was being published, Leopold Infeld found an error in Einstein's work causing Einstein to recant his argument, again allowing for the existence of gravitational waves (Cervantes-Cota, 2016).

The next major focus for physicists was detecting gravitational waves. As Einstein struggled with choosing a coordinate system to describe gravitational waves, other scientists were also not sure which coordinate system would be best used to detect gravitational waves. Scientists weren't sure how gravitational waves affected particles. The theoretical description of gravitational waves used units that were difficult to convert into observable quantities, but a scientists by the name of Pirani got around the conversion issue by showing that the tensor in Einstein's equation causes particles to vibrate back and forth in a quantifiable way (Cervantes-Cota, 2016). As can be seen in Figure 2., as a gravitational wave passes through an object, particles perpendicular to the gravitational wave are stretched and pulled. Imagine if a gravitational wave was passing into this sheet of paper. A ring of particles would cycle from a to d as shown below, where particles first pull away from the center of the ring, then push towards the center of the ring in an oscillating motion.

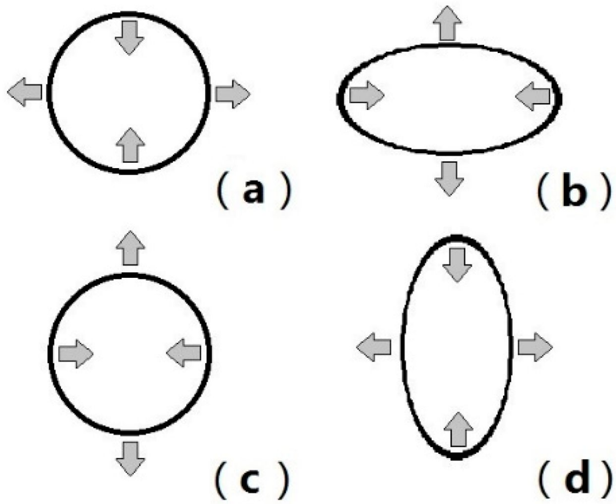

Figure 2. If a gravitational wave passed through the center of a ring of particles, the particles would be stretched in a perpendicular direction to the direction that the wave is traveling at in an oscillating motion (Cervantes-Cota, 2016).

At the Chapel Hill Conference of 1957, also known as the GR1 conference, leading physicists from around the world gathered together to discuss ideas of general relativity and gravitational waves. One of the most important results of the conference is called the “sticky bead argument” (Cervantes-Cota, 2016). As can be seen in Figure 3. the sticky bead argument essentially proves that gravitational waves have energy by arguing that gravitational waves can cause friction between particles. First, imagine a cylinder with two rings around the cylinder. If a wave passes perpendicular to the cylinder, the rings will be stretched and pulled along the cylinder. Because the rings and cylinder are both made of baryonic matter, they interact by the electromagnetic force, causing friction. Thus, the gravitational waves cause friction, a form of heat energy (Cervantes-Cota, 2016).

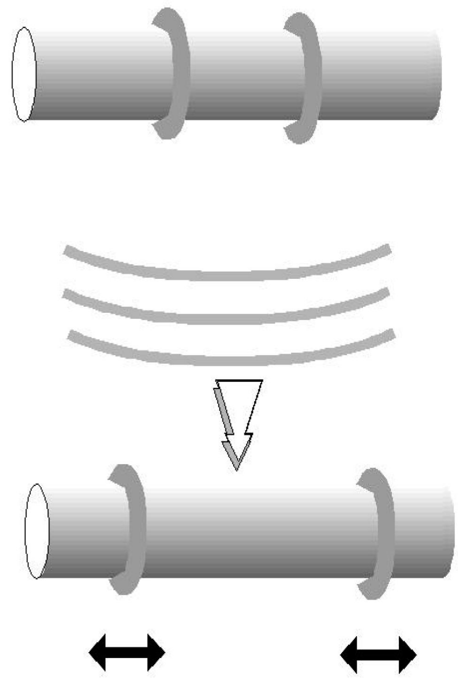

Figure 3. The sticky bead argument. The top picture shows a cylinder with two rings around it and the bottom picture shows a gravitational wave passing through the system, resulting in movement of the rings and thus causing friction. (Cervantes-Cota, 2016).

The first attempt to detect gravitational waves came from a scientist named Weber in the 1960's. The detection device was a 1.5 ton aluminum cylinder that hung by two wires in a vacuum chamber that had been cooled, all to reduce random vibrations from seismic events and thermal movement. The wires were connected to piezoelectric sensors that convert mechanical energy to electric signals. As gravitational waves moved through the cylinder, the cylinder would stretch slightly causing vibrations, but the effects of gravitational waves are significantly too small to be detected. Weber claimed to detect gravitational waves, but other scientists who replicated the experiment found no results (Cervantes-Cota, 2016).

Many physicists were pessimistic about gravitational wave detection and even existence until indirect evidence for gravitational waves came in 1978 with the observation of a pulsar binary system. A pulsar is a neutron star that emits electromagnetic radiation from its north and south poles. The pulsar is also wobbling on its axis, so its beam of electromagnetic radiation spins periodically. This beam hits Earth every 0.05903 seconds. While the period of the beam has held constant, the doppler shift, where the pulsar is moving towards and away from us as it rotates around its partner star, has shown us that the orbital period of the two stars has gotten smaller over the past four years. The reason for the smaller orbit is because some energy is radiated from the system in the form of gravitational waves. While the waves weren't directly detected, this does prove that gravitational waves do exist (Stannard, 2008).

One of the most important issues hindering the detection of gravitational waves using typical experimentation is that matter is held together by the electromagnetic force, which is much stronger than gravity, so the only real way to detect gravitational waves is to, instead of measuring the length of objects, measuring the distance traveled by light, because light always moves at a constant speed even if the distance traveled is altered. This leads us to the discussion of LIGO.

Detection at LIGO

The Laser Interferometer Gravitational-wave Observatory is composed of two main sites, one located at Hanford, Washington and the other at Livingston, Louisiana. Other sites were also built throughout the 2000's, mainly to run important tests to build full scale interferometers, including TAMA 300 in Japan, GEO 600 in Germany, and Virgo in Italy (Abbott, 2016).

An interferometer, first proposed in the 1960's and 70's, is composed of a laser source, a beam splitting mirror, two perpendicular arms, and a detector. A beam of light is emitted from a source that then reaches a mirror that splits the beam into two perpendicular beams. These two beams travel some distance down the length of two arms where they reflect off of a mirror and return to the beam splitter. Both beams are then reflected into a detector, where the beam's intensity is measured (Abbott, 2016). Because the distance in both arms of the interferometer is the same, the beams, after recombining and being detected, should be in phase, where the peaks and troughs of each wave line up and combine to increase the intensity of the beam. However, if a gravitational wave passed perpendicular to the interferometer, the length of one arm will be shrunk and the other arm will be lengthened, causing the beams to become out of phase, where the two beams interfere and decrease the intensity of the beam (Abbott, 2016). This is referred to as a Michelson interferometer.

This concept may seem simple, but gravitational waves have a very small effect on matter, so only the advanced LIGO sites, operational in 2015, were able to detect gravitational waves. Advanced LIGO has many enhancements from a normal Michelson interferometer, as can be seen in Figure 4. First, each arm length is 3km at one LIGO site and 4km at the other. Each arm also has a second set of mirrors that reflect the beam many times down the arm, amplifying the signal. The mirrors in the arms are kept at a resonance to amplify the signal by 300. A power recycling mirror at the input and at the output also increase the intensity of light. A 1064 nm Nd YAG laser is used to produce the beam and the beam is emitted at 20 W, amplifies to 700 W from the beam splitter, and is amplified to 100 kW by the arm cavities (Abbott, 2016). LIGO also uses suspension of the mirrors and materials that reduce thermal noise to decrease the effects of environmental and random disturbances. The site monitors for environmental disturbances that could affect results (Abbott, 2016).

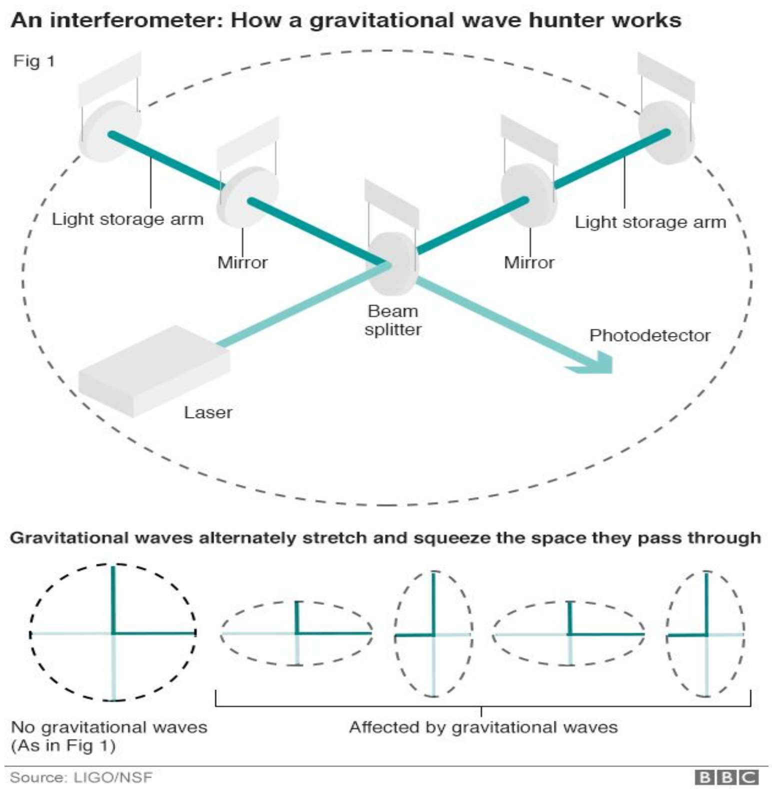

Figure 4. The top diagram shows the setup for Advanced LIGO. The diagram is similar to that for a Michelson interferometer except LIGO has an extra mirror in between the beam splitter and the end mirror for the arm (Amos, 2016).

The final result of this technology is that LIGO can detect length measurements as small as 1/10,000 the width of a proton (Kruse, 2016). The experiment measures the strain of the signal, which is the shift comparing the two beams, and a high strain indicates that the length of the arms of LIGO are being shrunk or expanded. From predictions of general relativity, certain patterns of strain are caused by gravitational waves from certain astronomical events.

GW150912 was the first detection of gravitational waves by LIGO, as can be seen in Figure 5. This event was the merger of two black holes of 36 and 29 solar masses (Izmailov, 2016). The final black hole had only 62 solar masses, so 3 solar masses worth of energy was radiated away in the form of gravitational waves. The event lasted for about 0.2 seconds and occurred 410 Mpc away from Earth. This means that the event occurred about 1.3 billion years ago (Kruse, 2016). The event matched the predictions of the merger of two black holes with a statistical significance of 5.1σ, strong enough to show conclusive evidence that the event fits our understanding of general relativity.

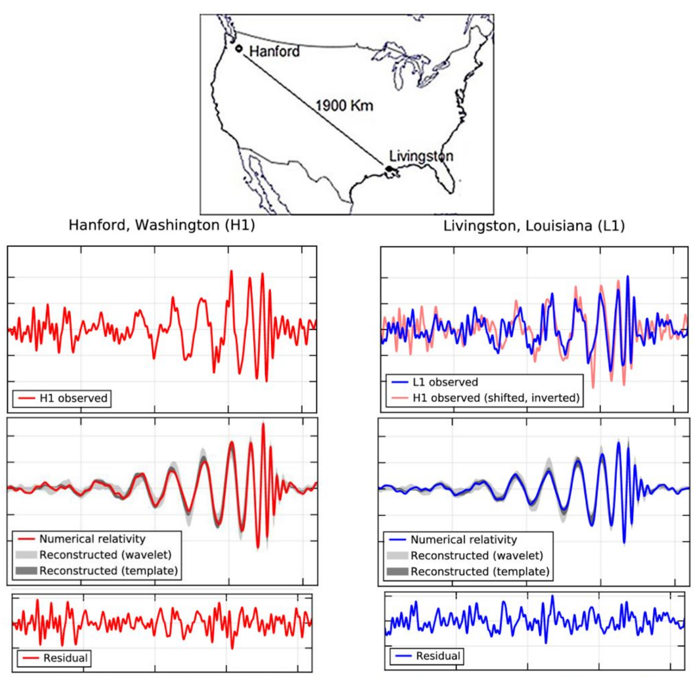

Figure 5. The graphs show the results of GW150914 recorded at both LIGO sites. The top graphs are the recorded events, the middle graphs are the expected graph for black hole merger, and the bottom graph is the random noise caused by thermal fluctuations (Cervantes-Cotes, 2016).

A second merger of black holes was detected in December of 2015 by LIGO. These detections were surprising to scientists who expected to detect the merge of neutron stars or a supernova explosion before black holes (Stannard, 2008). The reason that this data was determined to be from black holes is that the waves have an extremely high frequency, resulting only from very massive bodies orbiting at extremely close distances, closer than neutron stars could orbit (Abbott, 2016).

Conclusion

Although black hole merger was the first detection of gravitational waves, scientists are still searching for other signatures. Scientists also hope to be able to pinpoint the sources of gravitational waves by observing gamma ray bursts from black holes or neutron stars (Sathyaprakash, 2009). With other interferometer projects, such as an upgrade being completed to Virgo, scientists expect to be able to better observe and locate gravitational wave sources. A project is even being planned by the European Space Agency to build an interferometer in space, consisting of three satellites each over 5,000,000 km apart. The detection of gravitational waves has certainly set a new precedent in astronomy and will continue to result in a more developed understanding of our universe.

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