Game Theory of 2048

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I: Introduction and Background

The game 2048 is a strategic block-sliding game designed by Italian developer Gabriele Cirulli in the Spring of 2014. Although the game’s idea was not entirely original--there was a very similar game released a month prior to 2048 called Threes!--2048 was able to gain rapid popularity almost immediately following its debut because it was free to play, whereas its counterpart charged $1.99 to download and play. What’s even more impressive, Cirulli built the game from scratch in just one weekend as simply an amusing personal challenge to see if he could, never expecting for it to go viral the way it did (Wells). Little did he know, this little project would not only cause hundreds of thousands of people to download his game, but also spawn dozens upon dozens of spin-off games from the basic 2048, each with their own unique twist. All of this has resulted in making 2048 an iconic game of the year, whose craze has yet to die down over half a year later (Perez).

II: Rules

One huge contributing factor to 2048’s success is the simplicity of the game itself. The game is played on a 4x4 grid with sixteen squares, on which numbered tiles/blocks will slide. The entire goal of the game is to slide tiles of the same number together to combine them to reach higher and higher numbered tiles. And despite the name 2048, players may continue past the 2048 tile after reaching it, towards 4096, 8192, and beyond. Each turn, the player must pick one of the four cardinal directions to swipe, left, right, up and down, and all of the tiles on the board slide as far in that direction as they can, until they hit another tile or the edge of the board. If two tiles of the same value collide while moving, they combine into a tile the sum of the two smaller tiles. Furthermore, after each move, either a 2 or 4 tile randomly appears in one of the
remaining empty squares. The game ends when there are no legal moves remaining, which would mean that all sixteen squares have a tile and that no two adjacent tiles have the same value (Wells). These are simply the basic rules for the regular 2048; other variations of 2048 can be played on different sized boards, or with different tiles, or even in 3D. A quick summary of the game is provided below:

III: Basic Strategy of 2048

Contrary to most people’s initial beliefs when playing 2048 for the first time, there actually are strategies for getting high scores and even attaining the elusive 2048 tile, and simply swiping the tiles in random directions is most definitely not one of them. Before we get into the main strategy of the game though, it is important to make a simple observation about our objective. Reaching the 2048 tile requires the combination of two 1024 tiles—meaning we need to add up to 1024 twice AND we need them to be adjacent to each other in order to combine them. Furthermore, each 1024 tile requires two 512 tiles, which each require two 256 tiles, and so on and so on. All of this means that 1) it’s going to take a long time and a lot of tiles to get
even close to 2048, and 2) even as you grow your tiles, they will be useless and will just be taking up space unless they are next to each other and easily combinable (Zerman). This is where the main strategy comes into play. While there is no official name to this method, it involves playing the corners, and so for the remainder of this paper I shall refer to it as “cornering”. Cornering is basically the strategy of keeping your biggest tile in one of the four corners of the grid, and having the rest of your tiles decrease in magnitude the further they get away from the corner. Which corner you choose does not matter, so long as you pick one and stick with it throughout the game. Take a look at the following example:

![Tile Example](image)

Notice how the biggest tile (512) is in the top right corner, and how the tiles get smaller and smaller as you go down and to the left. To follow the cornering strategy is quite simple--rather than swiping in four directions, limit yourself to two directions for swiping so that your largest tile can hopefully never leave and expose its corner for a 2 or 4 tile to randomly appear and take its position. So if you pick the upper right corner, for instance, try to only swipe up and right, because swiping down or left may leave the corner exposed. There are several advantages to the cornering method which make it superior to simply swiping indiscriminately. First and foremost,
by keeping your biggest tile in the corner, it remains out of the way so that it can’t block other
tiles from merging. Your biggest tile will not have anything to merge with for the majority of the
time, since it will take a long time to build another tile of equal size to merge with it, so the big
tiles should stay to the sides and out of the way to let the smaller tiles interact and merge
together. Just look at the third panel of the first diagram above, which has a 2048 tile and a 128
tile in the middle of the grid. This is not an ideal position because both of these two large tiles
are blocking potential matches for smaller tiles (like the two 32 tiles). Secondly, by keeping your
bigger tiles in the same corner, you make it easy for them to connect and merge with each other
when necessary. Due to the aspect that every block must slide as far as it can for each swipe, it
can be extremely difficult to link up two specific tiles if they aren’t anchored in a corner. Even
still, cornering alone won’t guarantee getting you to the ultimate goal; fortunately, when
combined with a more advanced strategy, the chances of reaching 2048, and beyond, become
much higher.

IV: Advanced Strategy of 2048

As useful as cornering may be, it generally isn’t enough to get to 2048 by itself. Often
times, by only following the cornering method, you will eventually end up with a board similar
to the one below:
As you can see, the cornering method will lead to a lot of staggered potential matches that could eventually result in a deadlock. So instead of pure cornering, a much better strategy would be to combine it with another tactic, which I will designate as “snaking”, for reasons which become clear in a moment. The snaking method is basically a more advanced cornering method, because it still requires that you keep your highest tile in a corner. However, it deviates there because rather than have tiles diminish in value as they radiate away from the corner, the snaking method necessitates that your tiles diminish only along one edge from your selected corner, and upon reaching the next corner, snakes down one row and cuts back towards the opposite edge. Take a look at the following picture:

So in the picture, if we keep our largest block in the top right corner, the tile size should descend along the top row from right to left, and then drop to the next row and keep descending in value from left to right, and so on. Early on, this strategy won’t matter nearly as much, but especially later in the game when you’re trying to reach 2048 and beyond, this strategy is virtually the only consistent way of progressing onwards. The following series of diagrams will show a step-by-step example of how a successful snaking works:
From the top left panel, I swiped right to combine the 4 tiles, then up to combine the 8 tiles, then left to combine the 16 tiles, then left to combine the 32 tiles, then left to combine the 64 tiles, then up to combine the 128 tiles, then right to combine the 256 tiles, then right to combine the 512 tiles, and finally right to combine the 1024 tiles to make 2048. This example shows just how much strategy and careful placement of tiles is necessary to beat the game. It hopefully also sheds a little light on why I dubbed it the “snaking” method, due to the way each the blocks collapse into themselves in the zigzag pattern across the board. Furthermore, with the 2048 tile in the corner, you are ready to start over the snake again by building up a 1024 tile next to it, followed by a 512 tile, etc, so the snaking technique even resets itself for each new wave. Hence, the combination of the cornering and snaking methods is best way to achieve the evasive 2048.
V: Conclusion

Along with many other people around the world, I was swept up in the 2048 craze this past Spring, and I have yet to put it down for good. After hours upon hours spent playing this seemingly endless game, all I have to show for it is a personal record of reaching the 8192 tile twice ever. This got me to wonder how high would the largest theoretically possible tile could be, and it turns out that using the optimal strategy of cornering and snaking, the answer is easily intuitive. The highest tile possible given the limits of the board would be $2^{17}$, or 131,072. This can only be achieved if the board was set up as shown below:

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   16  32  4096  8192
    8  64  2048 16384
    4 128 1024  32768
   x 256  512  65536
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In the spot with an X, you would need to have a 4 tile spawn there to trigger the snake chain all the way to 131,072. If you spawn a 2 there, it would be game over. All this means is that I’ve got a long way to go before I can finally win and break free of the addicting game 2048.
Summary:

Despite being less than a year old, the block-sliding puzzle game 2048 has taken the world by storm and developed a global fanbase that rivals even the most popular games of the past decade. Even with such simple rules and repetitive gameplay, just one try is all it takes to see why 2048 is one of those games that you just can’t break free from. Because of the randomness of the location of each new tile after a swipe, this game can never be perfectly solved; however, general strategies have been developed to exponentially increase the chances of reaching and even surpassing the 2048 tile. These strategies have yet to be given official names, but based on how they look while being implemented, I’ve unofficially named them the “cornering” strategy and the “snaking” strategy. Essentially, these two tactics require that you keep your largest tile in a corner and never let it leave. The snaking method goes further and necessitates that you build your tiles along one edge in descending order away from your largest tile in the corner--basically in preparation for easily collapsing everything back to the corner to reach a new highest tile. A simple plan for a simple game, yet endless hours of fun--it’s no wonder why 2048 has captured our attentions for so long.
Bibliography


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