Final Exam, Math 89s
Game Theory and Democracy

Instructor: Hubert L. Bray

December 10, 2013

1	12	
2	15	
3	18	
4	10	
5	15	
6	30	
7	30	
8	10	
9	10	
10	10	
TOTAL	160	

Your Name:

Honor Pledge Signature:

Instructions: This is a three hour, closed book exam. The point values of each problem are indicated, and there are 160 total points on the exam.

No collaboration on this exam is allowed, but you may use a single $8\frac{1}{2} \times 11$ piece of paper, front and back, that you have prepared in advance. All answers should be written in the space provided, but you may use the backs of pages if necessary.

Express your answers in essay form so that all of your ideas are clearly presented. Partial credit will be given for partial solutions which are understandable. If you want to make a guess, clearly say so. Partial credit will be maximized if you accurately describe what you know and what you are not sure about. Good luck on the exam!

Problem 1. (12 points)
Suppose that in a preferential ballot election there are 3 candidates, A, B, and C, and that

- 10 voters rank the candidates A, B, C
- 5 voters rank the candidates B, C, A
- 6 voters rank the candidates C, A, B

on their preferential ballots.

(a) Briefly define Plurality Voting. Which candidate wins this election according to Plurality Voting?

(b) Which candidate wins this election according to Instant Runoff Voting, and why?

(c) Now suppose that 2 of the voters who had ranked Candidate A as their first choice decide that they do not like him after all and are allowed to change their votes to make Candidate A their last choice instead, so that now

- 8 voters rank the candidates A, B, C
- 7 voters rank the candidates B, C, A
- 6 voters rank the candidates C, A, B

on their preferential ballots. Now which candidate wins this election according to Instant Runoff Voting?

(d) Compare your answers to parts (b) and (c). Did anything weird happen as a result of 2 voters ranking Candidate A lower?

Problem 2. (15 points)
Determine the winners according to the following vote counting methods of the preferential ballot election with candidates A, B, C, D, E and margin of victory matrix

$$M = \begin{bmatrix} 0 & 7 & 11 & 13 & -8 \\ -7 & 0 & 9 & -5 & -6 \\ -11 & -9 & 0 & 14 & -10 \\ -13 & 5 & -14 & 0 & 12 \\ 8 & 6 & 10 & -12 & 0 \end{bmatrix}.$$

(a) Borda Count:

(b) Instant Runoff Borda:

(c) Worst Defeat:

(d) Schulze Method:

(e) Ranked Pairs:

Problem 3. (18 points) Determine the winners according to the following vote counting methods of the preferential ballot election with candidates A, B, C, D and margin of victory matrix

$$M = \begin{bmatrix} 0 & -1 & -2 & -3 \\ 1 & 0 & 4 & -5 \\ 2 & -4 & 0 & 7 \\ 3 & 5 & -7 & 0 \end{bmatrix}.$$

(a) Borda Count:

(b) Instant Runoff Borda:

(c) Worst Defeat:

(d) Schulze Method:

(e) Ranked Pairs:

(f) Which candidate is the LAST PLACE LOSER of the above election according to Worst Defeat?

Problem 4. (10 points) Essay Problem. Note that there are many correct answers. Credit will be awarded based on well reasoned arguments, so explain your answers precisely.

(a) In the election in the previous problem, which candidate do you believe most deserves to win and why?

(b) Which vote counting method do you generally like the most and why? Does your favorite method give the result you think is best in the previous problem?

Problem 5. (15 points)
Proof that Instant Runoff Borda is Condorcet. For this problem, assume that ties do not occur.

(a) Explain why the sum of all of the candidates' row sums of the margin of victory matrix must be zero.

(b) Prove that when there is a Condorcet winner (a candidate that beats all other candidates in head-to-head match ups), then this Condorcet winner is not eliminated in the first round of Instant Runoff Borda.

(c) Prove that Instant Runoff Borda is a Condorcet method.

Problem 6. (30 points) Determine the equilibrium strategies $p = (p_A, p_B, p_C, p_D)$ of each of the symmetric two-player zero sum games with payoff matrices listed below.

(a)

$$\begin{bmatrix} 0 & -3 & 5 & 4 \\ 3 & 0 & -2 & 3 \\ -5 & 2 & 0 & 5 \\ -4 & -3 & -5 & 0 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 0 & 5 & 13 & -7 \\ -5 & 0 & -5 & 3 \\ -13 & 5 & 0 & 10 \\ 7 & -3 & -10 & 0 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 0 & -3 & 1 & 7 \\ 3 & 0 & -2 & -10 \\ -1 & 2 & 0 & 8 \\ -7 & 10 & -8 & 0 \end{bmatrix}$$

(d)

$$\begin{bmatrix} 0 & 3 & -1 & -7 \\ -3 & 0 & 2 & 10 \\ 1 & -2 & 0 & -8 \\ 7 & -10 & 8 & 0 \end{bmatrix}$$

(e)

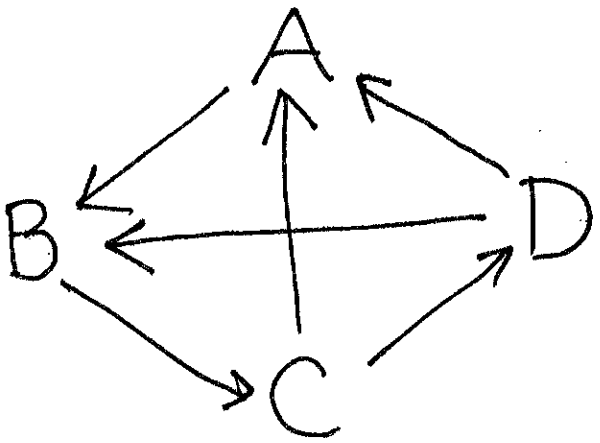
$$\begin{bmatrix} 0 & -1 & 3 & -5 \\ 1 & 0 & -2 & 4 \\ -3 & 2 & 0 & -5 \\ 5 & -4 & 5 & 0 \end{bmatrix}$$

(f)

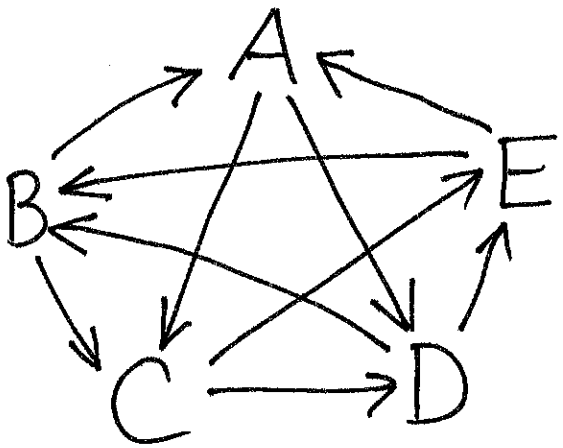
$$\begin{bmatrix} 0 & 3 & 5 & 4 \\ -3 & 0 & 2 & -3 \\ -5 & -2 & 0 & 5 \\ -4 & 3 & -5 & 0 \end{bmatrix}$$

Problem 7. (30 points) For each of the "Rock, Paper, Scissors" type games diagrammed below, find the equilibrium strategy $p = (p_A, p_B, p_C, ...)$ for the game.

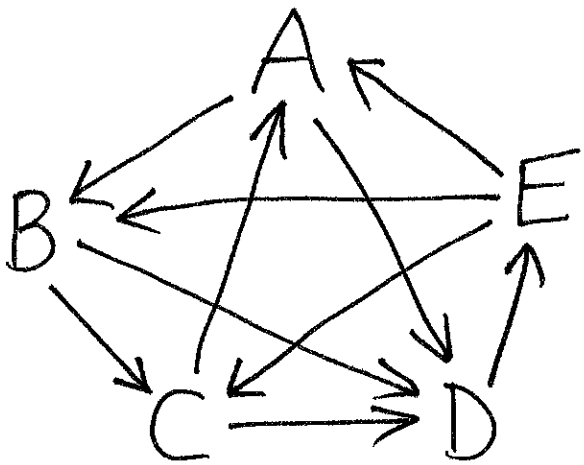
(a)



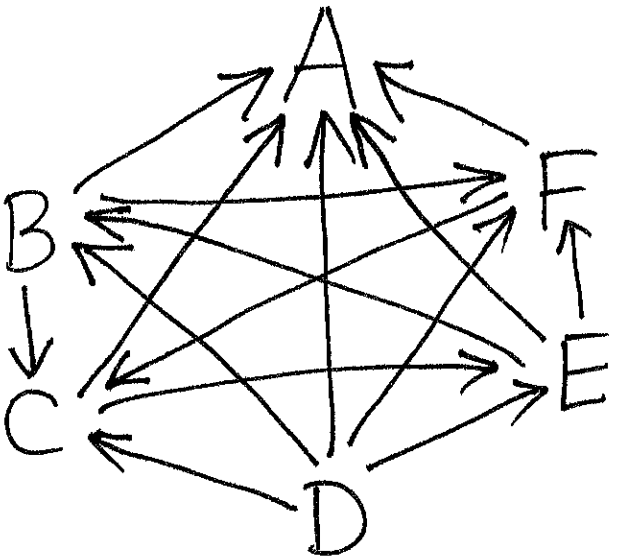
(b)



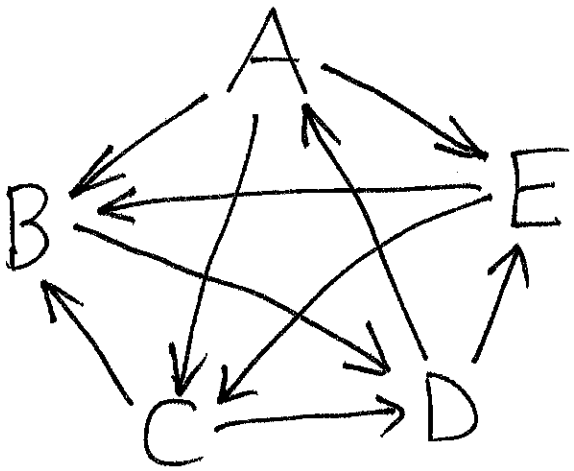
(c)



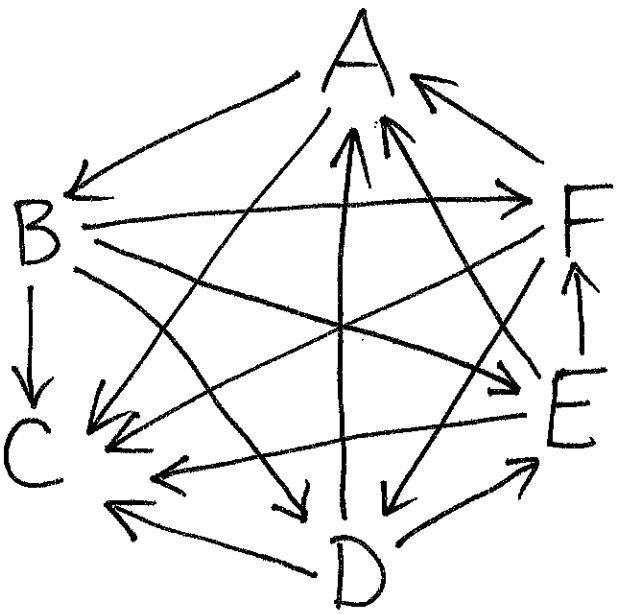
(d)



(e)



(f)



Problem 8. (10 points) In this problem, the quality of your comments, not the quantity, will be the basis for you grade.

(a) Summarize one of your favorite essays by one of your classmates.

(b) Discuss the essay. You may describe the ways you agree or disagree with the essay, add additional analysis, or provide any other comments you think are interesting.

Problem 9. (10 points) In this problem, the quality of your comments, not the quantity, will be the basis for you grade.

(a) Summarize another one of your favorite essays by one of your classmates.

(b) Discuss the essay. You may describe the ways you agree or disagree with the essay, add additional analysis, or provide any other comments you think are interesting.

Problem 10. (10 points) In this problem, the quality of your comments, not the quantity, will be the basis for you grade.

(a) In your opinion, what was the most interesting idea or topic discussed in the class?

(b) In your opinion, what was the most interesting topic not discussed in the class, or not discussed as much as you would have liked?