Game Theory and Democracy
Homework Problems

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Challenge Problem: Show that any voting model (such as the Unit Interval Model or the Unit Square Model) which has a point symmetry always has a Condorcet winner for every election, namely the candidate who is closest to the point of symmetry!

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Exercise: Show that if there are not any cycles of any length in the preferences of the voters, then there must be a Condorcet winner. (Don't worry about making your proof rigorous, just convince yourself of this fact, to the point you could clearly explain your ideas to someone else.)

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Exercise: Prove that the strongest chain strength of a chain from Candidate Y to Candidate X is 3. (If you can convince a skeptical friend that it is true, then that is a proof.)

Exercise: Prove that it is never necessary to use a candidate more than once to find a chain with the strongest chain strength between two candidates. Hint: Show that whenever there is a loop, like the one shown in red Y (7)▯ Z (3)▯ W (9)▯ Y (7)▯ Z (3)▯ W (11)▯ X, we can consider a shortened chain by simply deleting the red loop Y (7)▯ Z (3)▯ W (11)▯ X which is at least as strong as the original loop.

Exercises: Prove that all of the chains above are in fact strongest chains, as claimed. Hint: The next figure, which has all of the information of the margin of victory matrix, will be useful, and is the type of figure you may choose to draw for yourself for other elections, from time to time.

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Exercise: Show that every candidate chain ties with themselves.

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Exercise: Show that, according to Worst Defeat, the winner of the above election is Candidate S.

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Exercise: Show that, according to Worst Defeat, the winner of the inverted election and hence the last place loser of the original election is also Candidate S.

Exercise: Show that, according to Worst Defeat, the winner of the original election after the last place loser, Candidate S, is removed is Candidate R.

Challenge Problem: Prove that, according to the Schulze Method, the winner and the last place loser of an election are never the same candidate. Hint: Whereas the winner chain beats every other candidate, show that the last place loser must chain lose to every other candidate, and hence cannot be the same candidate.

Exercise: Show that, according to the Schulze Method, the winner of the original election is Candidate S.

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Exercise: Show that, according to the Schulze Method, the winner of the inverted election (shown above) and hence the last place loser of the original election is Candidate U.

Exercise: Show that, according to the Schulze Method, the winner of the original election after the last place loser, Candidate U, is removed is Candidate R.

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Exercise: Show that the last place loser of the above election, according to Ranked Pairs, is Candidate U. Hint: By definition, the last place loser is the winner of the election when all of the voters' rankings of the candidates are reversed, which does not change margins of victory at all, other than switching the winner and the loser of the head-to-head matchups. Hence, the "priority of greatest majorities" procedure above will be exactly the same as before, except for producing the exact reverse overall ranking.

Exercise: Show that, according to Ranked Pairs, the winner of the election after the last place loser, Candidate U, is removed is still Candidate R.

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Exercise: Show that, according to Ranked Pairs, Candidate Z is the last place loser of this election.

Exercise: Show that, according to Ranked Pairs, Candidate R still wins the election after the last place
loser, Candidate Z, is removed from the election.

Exercise: Show that, according to Worst Defeat, Candidate S is the winner of this election.

Exercise: Show that, according to Worst Defeat, Candidate S is the last place loser of this election.

Exercise: Show that, according to Worst Defeat, Candidate R wins the election after the last place loser, Candidate S, is removed from the election.

Exercise: Show that, according to the Schulze Method, Candidate S is the winner of this election.

Exercise: Show that, according to the Schulze Method, Candidate Z is the last place loser of this election.

Exercise: Show that, according to the Schulze Method, Candidate R wins the election after the last place loser, Candidate Z, is removed from the election.