

Math 108 Exam 1

Name: _____

I have neither given nor received any unauthorized help on this exam and I have conducted myself within the guidelines of the Duke Community Standard.

Signature: _____

Instructions: You may not use any notes, books, calculators or computers. A box is provided for your answer you must write your answer (and nothing else) in the box to receive full credit for the problem. Even if the correct answer appears somewhere else on the page, you will not receive full credit. Moreover, you must also **show the work** you did to arrive at the answer to receive full credit. If you are using a theorem to draw some conclusions quote the result. You have 75 minutes to answer all the questions.
Good Luck !

Question	Max Point	Score
1	20	
2	20	
3	20	
4	20	
5	10	
6	10	
Total	100	

1. (a) (10 points) Solve the equation:

$$ty' + 2y = t^2 - t + 1$$

Answer:

(b) (10 points) Solve the initial value problem:

$$\begin{cases} ty' + 2y = t^2 - t + 1 \\ y(1) = 5/12 \end{cases}$$

and determine where the solution is defined.

Does this contradict the existence and uniqueness theorem for first order linear differential equations?

Answer:

2. (20 points) Solve the initial value problem:

$$\begin{cases} y' = y^2(2 + 2x) \\ y(0) = y_0 \end{cases}$$

and find the domain of definition of the solution.

Answer:

3. Consider the equation:

$$(xy^2 + bx^2y) + (x + y)x^2y' = 0$$

- (a) (10 points) Determine for which value of b the equation is exact and solve for the equation for this value of b .

Answer:

- (b) (10 points) Solve the equation for $b = 2$ and the initial condition $y(0) = 0$. (Hint: don't try to use the integrating factor method, think about a technique you practice in the homeworks)

Answer:

4. (a) (5 points) Check that x is a solution for the equation:

$$x^2 y'' - x(x+2)y' + (x+2)y = 0$$

for $x > 0$

Answer:

(b) (15 points) Find a second solution of the equation:

$$x^2y'' - x(x+2)y' + (x+2)y = 0$$

and prove that the two solutions are linearly independent.

Answer:

5. (10 points) Sketch the graph of the solutions of the equation:

$$y' = -y(3 - y)(5 - y)$$

and classify the constant solutions.

Answer:

6. (10 points) Determine in which region of the (x, y) plane the existence and uniqueness theorem applies for the equation:

$$(y - x^2)y' - \ln(x^2 + y^2 - 1) = 0$$

Answer: