AN OBJECTIVE APPROACH TO STUDENT-CENTERED INSTRUCTION.

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ABSTRACT:

We describe an algorithm for planning lessons based on the identification and achievement of student learning objectives. We discuss ways in which this algorithm can help novice instructors to include opportunities for active learning in their lessons. Examples of mathematics lessons are included to illustrate the use of the algorithm. Reactions of novice instructors to the algorithm are described, and ideas for incorporating this algorithm into an instructor training program are offered.

KEY WORDS: Lesson planning, student-centered instruction, novice instructors, instructor training, learning objectives, active learning.

1. INTRODUCTION

The primary purpose of this article is to describe an algorithm for designing college-level mathematics lessons. The approach that we describe is purposefully designed to be compatible with ideas of student-centered instruction that have gained popularity in introductory college mathematics courses.

Since the 1986 Tulane conference [17], there has been a serious effort in the college mathematics community to address perceived problems with undergraduate mathematics education. One part of this effort has been to encourage mathematics instructors to use student-centered instruction (SCI) methods in addition to, or in place of, lecturing [1, 4, 9, 15, 18, 20, 22, 30, 31, 33, 39, 41, 46, etc.].

Many innovative college teachers feel that SCI represents an excellent collection of methods for creating learning environments where students are encouraged to learn actively in the classroom, and to continue learning outside of the classroom. However, implementing SCI appears to be either practically or ideologically very difficult for many college mathematics instructors. DeLong and Winter [15] have described some of these practical and ideological difficulties in elementary college mathematics courses. There is some evidence to suggest that these difficulties are not confined to SCI in mathematics. Felder and Brent [21] discuss concerns of faculty who have tried various

aspects of SCI in undergraduate engineering courses. Trigwell [44] reports similar difficulties with faculty development in the context of physics and chemistry.

Student-centered instruction is both a collection of assumptions about the purpose of instruction and a collection of instructional techniques. Felder and Brent [21] characterize student-centered instruction as follows.

> "SCI is a broad approach that includes such techniques as substituting active learning experiences for lectures, holding students responsible for material that has not been explicitly discussed in class, assigning open-ended problems and problems requiring critical or creative thinking that cannot be solved by following text examples, involving students in simulations or role-plays, assigning a variety of unconventional writing exercises, and using self-paced and/or cooperative (team-based) learning. In traditional instruction, the teacher's primary functions are lecturing, designing assignments and tests, and grading; in SCI, the teacher still has these functions but also provides students with opportunities to learn independently and from one another and coaches them in the skill that they need to do so."

Clearly, instructors' views of how mathematics should be taught have a significant effect on the forms of classroom activity that the instructors plan for their classrooms [26, 43, 44]. A synthesis of several different sources [15, 21, 44] suggests that instructors who employ methods of SCI need to experience a shift to what Kuhs and Ball describe as either the "learner-centered" or "classroom-centered with emphasis on conceptual understanding" views of how mathematics should be taught. Accompanying this change in views of how mathematics should be taught will be a period where instructors gain expertise in the classroom procedures necessary to implement techniques of SCI.

Descriptions of this process that have been contributed to the literature include taxonomies of the stages that typify instructors' development in conceptions of learning and teaching [26, 43, 44]. Other work describes developmental stages that novice instructors appear to pass through as they gain facility with the use of different teaching techniques [34]. Still other work has examined areas of resistance to using the techniques of SCI [21] and the difficulties that novices encounter when trying to develop, learn, or perfect classroom procedures for having the students engage in learning activities [15]. One area that has received little attention is how college-level instructors plan lessons in which they expect to use techniques of SCI, and how their assumptions about teaching and learning affect their lesson planning processes (see Section 2). Understanding how instructors plan their lessons is of critical importance, as research indicates that most of the decisions that instructors make that substantially affect the quality of the learning environment are made during the "pre-active, planning phase" [6].

Much of the work on how to plan for the integration of SCI techniques into mathematics lessons that has appeared encourages instructors to choose one particular technique (for

example one particular strategy of cooperative learning such as "think-pair-share") and to consciously employ the selected technique at some point during their otherwise conventional lectures [18, 33, 39]. The problem that we see with such advice is that while instructors are encouraged to use student-centered instructional methods, there is little or no attention paid to the instructors' conceptions of learning and teaching. As documented by DeLong and Winter [15], while novice instructors will sometimes attempt to use SCI methods, they often do so with no clear purpose or objectives, or else little belief that learning activities are worth the class time devoted to them. Such instructors typically view mathematics as a subject that is best taught via fast-paced, instructor-centered lectures, a view that is obviously incompatible with the instructional techniques of SCI.

Many studies demonstrate that novice instructors usually replicate the lesson structure, approach to the subject matter, and learning environment in which they first encountered the particular subject as students themselves [3, 15, 16, 18, 27, 43]. As noted in some of these studies [15, 18], this will usually be an instructor-centered lecture. In a recent article, we describe the lesson planning behavior of a cohort of novice college mathematics instructors [16]. In that article it was documented that the novice instructors tended to

1. conceptualize the task of planning a lesson as the organization of a unit of content for presentation,

2. regard the objective of a lesson as "covering" the appointed content of subject matter,

3. instinctively use, or seek, little or no knowledge of the students in the class as learners of mathematics,

4. imagine that the process that they will engage in is one of transmission of factual information that students will learn by correctly receiving the transmitted information,

5. plan for little or no classroom activity that presents opportunities for instructors to accurately assess students' learning,

6. treat the subject matter in much the same way as the novice instructors first learned the material as students themselves, and

7. reproduce the classroom environments in which the novice instructors first learned the material as students themselves.

We suggest that these views of mathematics teaching and learning are potentially problematic, especially for novice instructors attempting to implement SCI. In order for novice instructors to have a significant opportunity to successfully implement methods of SCI, they need to be introduced to lesson planning practices that stimulate them to break

with the methods of instruction and lesson planning that they experienced as students themselves. As Steven Krantz [25, p. 112] has noted:

> "If you simply tack new teaching techniques on to your existing lectures and problem sessions then those new techniques probably will not succeed. If you want to use the new techniques then—to some degree—you will have to learn to teach anew."

In this spirit, the primary purpose of this article is to describe an approach to lesson planning that creates opportunities for instructors to include SCI more naturally and purposefully in their lessons than, as Steven Krantz put it, "tacking new techniques on to existing lectures." The approach intentionally leads instructors to plan lessons focused on student-centered learning objectives, rather than merely focused on the content and performance.

For instructors who are interested in trying this approach to lesson planning, we provide examples of mathematics lessons that were planned using the lesson planning algorithm in order to illustrate its use. For those who have the responsibility for training new college mathematics instructors, we suggest that this algorithm can be introduced as part of a training program for novice instructors. Finally, we describe some of the reactions of novice instructors to the algorithm, and we list ways in which the lesson planning practices of novice instructors using the lesson planning algorithm differ significantly from the kinds of limiting practices we have previously described [16].

2. A BRIEF REVIEW OF RESEARCH ON TEACHER PLANNING

Teacher planning has been studied in the context of elementary, middle, and secondary school instruction. As noted by Sardo-Brown [37], however, relatively few studies have been completed, and those that have been usually include few subjects. There appears to be very little literature describing or discussing the planning processes of college-level instructors. (We have recently made a contribution that studies the planning processes of a cohort of novice college mathematics instructors [16].) Most of our discussion here will, therefore, deal with research conducted with school teachers.

Teacher planning has been conceptualized as a "psychological process which allows the teacher to visualize the future [of the classroom environment] and construct a framework to guide his or her future actions" [5, 13]. According to Clark and Yinger [12], teachers typically engage in eight different types of planning. These forms of planning include: weekly, daily, unit, long-range, lesson, short-range, yearly, and term. Yinger [47] and Sardo-Brown [36] observed in the context of elementary and middle school instruction that these forms of planning were nested, in the sense that the daily planning took place in the context of weekly planning, which took place in the context of units planning, etc. Here we note a significant difference between school planning procedures and the situation for college instructors teaching multi-section service courses. Namely, college

instructors typically do not need to make a great deal of planning decisions beyond the context of the individual lesson. This is because large, multi-section college mathematics courses are typically taught from a common syllabus that prescribes the curricular unit that each is instructor is expected to teach on any given day of the course. With this understanding we concentrate on the specific area of lesson planning, because of the eight forms of teaching planning identified by Clark and Yinger, this seems to be the most relevant to introductory college mathematics courses.

Models for planning individual lessons are nothing new. Teaching methods textbooks aimed at elementary and secondary education majors include very thorough descriptions of models for planning lessons [7, 8, 14, 19]. The basis for these models is arguably the "Objectives First" or "Rational" model first proposed by Tyler in 1949 [45]. This is a linear model that consists of four steps:

1. specify objectives,

2. select learning activities,

3. organize learning activities, and

4. specify evaluation procedures.

During the 1980's, the accuracy and relevance of this model were challenged [10] on the basis that planning is usually not a linear process, and that there is considerable evidence to suggest that teachers do not follow an "objectives first" approach, but instead tend to simply search for activities that will use all of the instructional time available [36, 47, 49]. An alternative model was proposed by Clark and Yinger [11] that described lesson planning as a three-stage process of problem solving. Briefly, these steps are as follows:

1. the teacher forms an initial idea of an activity that may be suitable for the class,

2. the teacher elaborates on the initial idea, and

3. the teacher implements the activity.

While this model does appear to represent teachers' actions more accurately (especially the actions of those who eschew the "objectives first" approach and select activities to fill up class time), this model is probably not entirely satisfactory as a framework for novice teachers to employ when planning their lessons.

One of the most frequently exhorted ideas in teaching and instructor training is that careful and thorough preparation or planning for a lesson makes the lesson more valuable and productive [25]. In an expert-novice study of school teacher planning, Leinhardt [29] reported that "consistent but flexible lesson structures" was one of three important criteria distinguishing experts from novices (the other two being rich agendas and clear

explanations). A failure to adequately plan is frequently seen as a contributing factor to unproductive lessons [25, 48]. Paradoxically, in the college context with the constraint of a limited number of contact hours and the accompanying need to make lessons as valuable and productive as possible, instructors typically receive no exposure to models for lesson planning, and little advice for constructing their lessons. As noted by Smith [41], "We take for granted that anyone who has mastered the subject at this level [holding a Ph.D. in mathematics] is prepared to teach."

To situate the present work in terms of what has already been done, we observe that the approach to lesson planning that is described herein is not intended to be a model of instructor behavior in the sense of accurately representing what many college mathematics instructors actually do when they plan their lessons. As discussed in the introduction, the natural lesson planning practices of novice mathematics instructors are typically not at all like the process we describe in Section 4. Instead, our approach represents a flexible framework that encourages instructors to substantially incorporate their knowledge of students and of pedagogical techniques into their lessons at the planning stage. Our approach encourages instructors to consider assessment of student learning as an important part of their activities within the classroom, rather than something that they either ignore altogether, or else make up "on the spot."

Our approach to lesson planning is a synthesis of the "Objectives First" model of Tyler, [45], the problem-solving approach of Clark and Yinger [11], and the Shannon-Weaver model for telecommunications [40]. Our procedure is non-linear in that we have modified Tyler's model by the deliberate inclusion of feedback loops within the planning procedure (see Figure 1), thereby incorporating the conclusion of Clark and Yinger [11] that planning processes are typically non-linear in character. In order to make the procedure straight-forward and easy to use for novices, sub-parts are still reasonably linear in character. Like the Tyler model, our approach emphasizes the formulation of learning objectives as a preliminary step.

Lesson planning procedures (most notably those for schools advocated by Madeline Hunter) have been widely criticized for their inflexibility and their incompatibility with some instructional objectives [2, 23, 32]. We wish to stress that we do not see or advocate our procedure as a definitive series of steps that must be undertaken in the prescribed order according to rigid and inflexible rules. Rather, we believe that our procedure represents a framework for organizing thought about lesson planning that helps instructors to utilize their knowledge of students and pedagogical techniques as well as their knowledge of mathematics and curriculum. As is obvious from Section 4, this procedure requires instructors to reflect on both their instructional practice and their plans, and to make a lot of decisions for themselves. Our experience indicates that this emphasis creates the opportunity for instructors to enjoy a great deal of flexibility when planning lessons using this framework, which is evidenced by the instructor comments included in Section 6.

3. THE CLASSROOM ENVIRONMENT WE ENVISION

We have both taught, and trained other instructors to teach, in a style that emphasizes active student participation in class and cooperative group activities both inside and outside of class. The ideas about teaching, classroom management and classroom organization on which this approach is based have previously been described [15, 35, 42]. The text used in such a course is one aligned with the "reform" or "renewal" movements in undergraduate mathematics. For example, in our calculus classes the text used was by Hughes-Hallet, Gleason, et al. [24]. Students typically have access to technology in the form of hand-held graphing calculators.

A model class period might consist of several mini-lectures interspersed with individual, pair, or group-based activities. The mini-lectures are usually no longer than ten minutes in duration. The results of activities are often reported and explained to the rest of the class by students, rather than by the instructor. Establishing the mathematical validity of results is usually the job of the students, rather than the exclusive domain of the instructor.

The instructor's role in this teaching method is a facilitator and guide for student learning. During in-class group work, students typically spend time during the lesson working on problems from the course text in groups of three or four. While the students work, the instructor circulates and tries to clarify specific issues.

Students in these classes typically participate in homework groups, where teams of three to five students would meet once or twice per week outside of class time to complete four challenging, non-routine problems. Students are also expected to regularly read their textbooks as part of their preparation for class, although the students are not expected to completely comprehend everything that they read on their first encounter with the material.

4. A FRAMEWORK FOR LESSON PLANNING

In this section, we briefly describe the concept of a learning objective, and offer some examples to clarify the notion of a learning objective in the context of an introductory college mathematics course. We then present an algorithm for planning student-centered lessons that relies heavily on the identification of learning objectives.

4.1 Learning Objectives

We begin with a working definition, in order to differentiate learning objectives from content.

Farrell and Farmer define an *instructional objective* to be: "...a statement that describes a desired student outcome of instruction in terms of observable performance under given

conditions" [19, p.196]. There are two important aspects of an instructional objective to note. First of all, an instructional objective describes student outcomes of instruction. Opposed to this would be a content objective, such as: "Today I will teach the chain rule," which describes something that the instructor intends to do (teach) and the content area (the chain rule). There is no mention whatsoever of the students, let alone the outcomes of instruction for the students. Second of all, an instructional objective is stated in terms of observable performance. Opposed to this would be an objective that is either not observable, or not stated as such. For example, "Today the students should learn how to use the chain rule," would not be sufficient as an instructional objective as stated. If the statement was more specific on what was intended by the verb "learn," and how the aspects of this "learning" might be observable, then the statement could represent an instructional objective. An example of a set of closely related instructional objectives would be:

"Given three functions, only one of which is a composite function, the students should be able to:

(a) determine which function is composite,

(b) to determine what functions were composed to make that function, and,

(c) to correctly formulate the derivative of that function using the chain rule."

This objective could be assessed by the instructor during a lesson by giving the students such a set of three functions, and by asking them to work individually or in small groups on meeting the objective.

In this article, we have chosen to refer to objectives with these qualities as *learning objectives*, rather than instructional objectives to emphasize the importance of focusing on the students' learning of mathematics, rather than the instructor's presentation of mathematics.

4.2 The Lesson Planning Algorithm

In this section we describe the lesson planning algorithm. Figure 1 shows schematic representation of the algorithm. Following Figure 1 is a set of instructions for implementing each step.

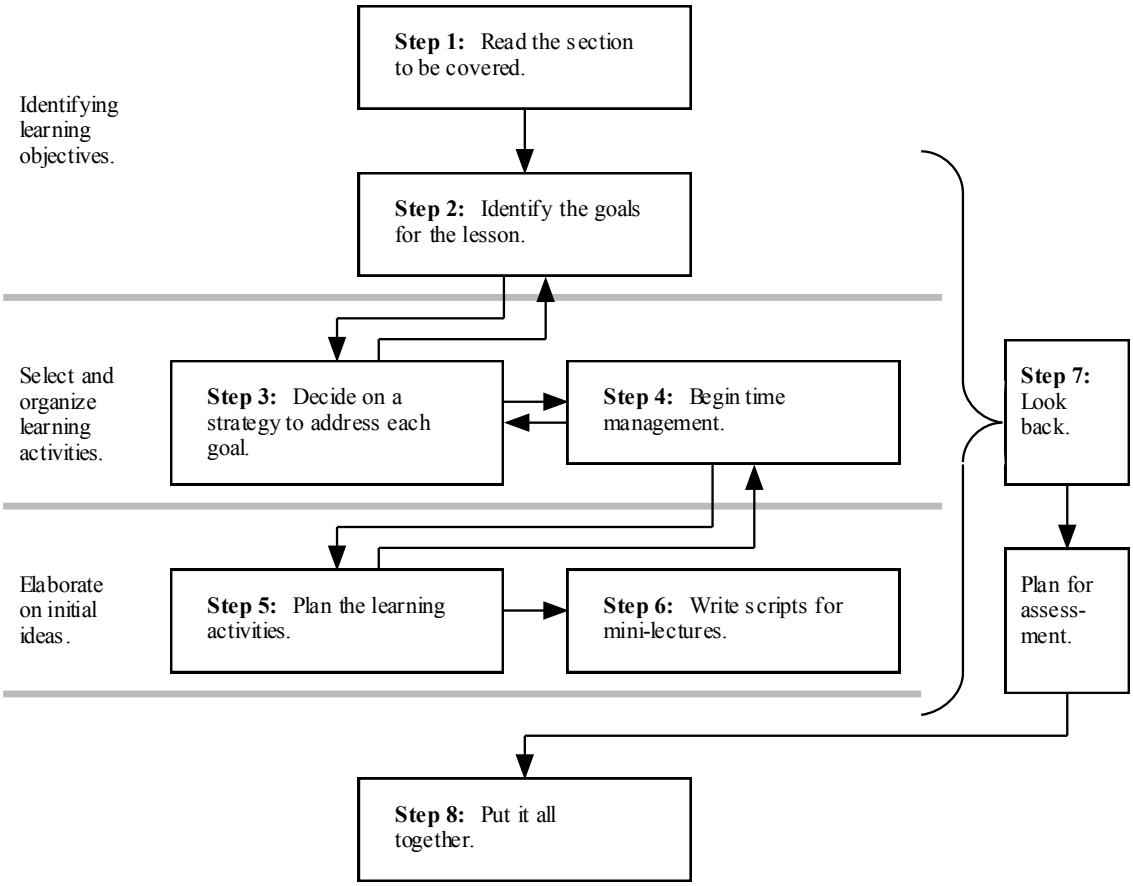

Figure 1: A Schematic Representation of the Major Steps in the Lesson Planning Algorithm

Step 1: Read the section to be covered.

Typically, you will not need to read the section in as painstaking depth as you do your own textbooks, or as you hope your students will do with this book. However, it is very important that you know what topics are treated in the section and how the book treats them. You will plan your lesson under the assumption that your students have read the section. This is impossible to do unless you have read the section yourself.

Step 2: Identify the learning objectives for the lesson.

After you have read the section, jot down the 3-5 main ideas that the book presented. Add any other main points that you want to address that the book did not.

Once you have identified the main ideas from the section, use them to formulate the 3-5 student learning objectives for the class session. These objectives should state what bits of knowledge and what skills students should have acquired by the end of the lesson. Make sure that the objectives focus on the students' learning, and not instructor-focused.

This will help you to concentrate on their learning, and not on you performance. Be careful to state these objectives in a way that will allow you to observe whether or not the students have actually achieved each objective.

Step 3: Decide on a strategy to address each goal.

In an interactive classroom you will seek a balance between lecturing and student-centered activities. A model lesson will likely consist of an interplay between short mini-lectures and activities that either foreshadow or solidify the ideas addressed in these mini-lectures. After identifying the learning objectives, you will want to decide which of these will be best addressed through an instructor explanation, which will be best addressed through a student activity, and which might require both. Keep in mind that a mini-lecture can include some student participation, and that an activity will almost certainly include some instructor explanation, so these strategies are not mutually exclusive. Look back at your goals from Step 2, and identify what type of strategy, or strategies, you will use to address each by marking "lecture," "activity" or "both."

Step 4: Begin time management.

Now that you know very roughly what you will be doing, you need to decide on a time frame in which to work. Identify what administrative issues (such as the return or collection of students' work, reminding students of class policies, alerting students to important course events) that you have on your agenda, and how long they will probably take. Identify the intended length of each of your mini-lectures by dividing them into five minute lectures and ten minute lectures. To help students maintain a high degree of mental activity and attentiveness for the entire lecture, do not lecture for more than 10 minutes at a time. Finally, subtract the lecturing total from the total class time to find the amount of time remaining for student activities. Allocate time for each activity that you have planned.

Step 5: Planning the student activities.

Now you will decide what activities the students will do in the lesson. Recall from Step 3 how many activities you hand in mind, and from Step 4 how much time you have at your disposal. Choose an activity for each student learning objective from Step 3 that requires an activity. To begin with, it might be easiest to choose a problem out of the textbook that addresses the stated learning objective. At other times you might design your own activity by making up a problem, designing a worksheet, or creating some other activity that reinforces a skill (e.g. roundtable activities or checking each others' individual work in pairs). The latter is more interesting and rewarding, but also more difficult. Try to vary using activities done in groups, in pairs, and individually. Write down the activities that you chose for this section, and whether you will have the students do them in groups, pairs or individually.

Decide how long to allocate for each activity. Group activities often take longer than you hope, but having a particular time-frame that you clearly communicate to the students can help you to manage time efficiently. Make sure you allocate approximately 2 minutes to the set-up and wrap-up of each activity. When looking at problems that are especially difficult, or that have many parts, it is often easier to plan and manage the time if you break the activity into pieces. For example, you might allocate 4-5 minutes for parts (a) and (b) of a problem, and then take 2 minutes to discuss these parts with the class. After this you would have the class continue with parts (c) and (d), etc. Write down how long you expect to spend on each of the activities for this section, and make notes on how you might break down the longer activities.

Finally, make notes on the particular execution of each student activity. Some considerations are: (1) what you will say to set up the problem, (2) the mechanics of getting the students into groups and working, (3) how you will ask for the results to be reported, and (4) what you will say to emphasize how the main points of the activity relate to the previous or upcoming mini-lectures.

Step 6: Writing the mini-lectures.

Go back to Step 3 and recall the objectives you plan to address via mini-lectures. Write out a brief script for each mini-lecture on note cards or paper that you can carry around with you. When writing your scripts

- anticipate students' difficulties with the material and how you will help them (plan how you will simplify the language, and how you can illustrate or convey your intuitive sense of what is going on),

- move from concrete to abstract,

- plan what you will write on the board, especially in the way of examples and graphs,

- decide on a higher-level question that you can ask your students during the mini-lecture, and,

- plan what you will say during the transition from the lecture to an activity or from the lecture to another lecture.

Step 7: Looking back.

Refer back to the 3-5 learning objectives that you identified in Step 2. Now you will plan how to briefly assess the effectiveness of your chosen strategies in meeting these learning objectives. If you have been properly facilitating the group activities, then you should already have an idea for how well the students are meeting the learning objectives. You need to also identify how you will assess their understanding of the mini-lectures and individual activities that you have given them. Some brief assessment techniques that

can be used during class are (1) to call on a few students with questions, (2) to give the class a question and ask them to vote on the answer, or (3) to have them try a practice problem that they trade with their neighbor, and then find out what portion of the students successfully completed the problem. You might also consider an overall assessment of the class period. As an example of this kind of assessment, you might end class by collecting a "minute paper," in which you ask students to summarize what they learned, to tell you their muddiest point, or to do a brief exercise.

If you find that students are not succeeding in obtaining the skills or knowledge that you intended them to, then you will have to decide how to alter the course of your plans in order to revisit the issue, perhaps in a new way. Adjusting your plans "on the fly" is, of course, a difficult proposition. However, it is a skill that you should work to acquire as you become more experienced.

Step 8: Putting it all together.

Write down an outline of the order of the day's administration, lectures and activities. Finally, jot down how you will wrap up the class period. Try to regularly include a short summary of what the students learned that day or a preview of the upcoming reading.

Final note: It is better to over-plan than to under-plan.

Often you will plan too much, and you will be unable to get through your entire day's lesson. In these situations, assign activities that you were unable to get to as individual homework, or resort to emergency lecturing if you get extremely behind. As you gain experience, you will hone your sense of how much to include in a class period.

5. EXAMPLES

In this section we provide examples of lessons that were planned using the lesson planning algorithm described in Section 3. We note that the utility of this approach to planning lessons is not confined to elementary mathematics, and it has been used successfully in more advanced classes, such as applied mathematics, complex analysis and partial differential equations.

5.1 Example 1: Separable differential equations and equilibria on slope fields

Course: Calculus 1 or 2. (Note: All references to the textbook refer to Hughes-Hallett, Gleason, et al. [24].)

At this point of the semester, students had learned differential calculus, and had been introduced to antiderivatives as the opposite of derivatives. Students had not been taught any techniques of antidifferentiation, although many students were familiar with this from AP calculus classes. Students were encouraged to find antiderivatives by making

educated guesses and then differentiating to check their guess. Students had encountered constant-coefficient differential equations of the form:

$$\frac{dy}{dt} = ky \qquad \text{and} \qquad \frac{dy}{dt} = ky + b\,.$$

Students had solved the first differential equation by guessing and checking, and were shown how to perform a substitution to reduce the second equation to the first. Students had encountered more complicated differential equations and had used Euler's method to find approximate solutions to initial value problems.

Learning Objectives:

1. Students can articulate that the idea of the technique of separation of variables is to separate independent and dependent variables so that the differential equation may be solved by taking antiderivatives.

2. Students can separate the variables in a given differential equation.

3. Having separated a differential equation, students can antidifferentiate and perform the algebra to find a formula for the solution of the differential equation.

3. Students are able to calculate the equilibrium solutions of a differential equation without having to explicitly solve the differential equation.

4. Given a differential equation that models a phenomenon, students can interpret the significance of the equilibrium solutions.

Lesson Plan:

00:00-00:10 Introduction: Remind students of upcoming test (to be held in lab next Tuesday). Take questions on the homework. Use the following examples to remind students of the methods for solving differential equations that the students have encountered so far.

Example 1: $\frac{dy}{dt} = 3y$ (Type 1 differential equation)

Example 2: $\frac{dy}{dt} = 3y + 1$ (Type 2 differential equation, "Phantom Constant" or z-substitution)

Example 3: $\frac{dy}{dt} = 3t$ (Antiderivatives)

00:05-00:15 Activity: Form the students into pairs, and have them attempt problem 33 from page 509 of the book (i.e. solve dy/dx = x/y). (The idea here is for the students to

try the methods that they know, and then realize that none of these methods work.) If any pair works out how to solve the problem, then have them write their solution up on the board for the rest of the class. Have any early finishers write their solutions on the board, or else have them start working on problem 34, page 509.

00:10-00:15 Mini-lecture: Either comment on the solution on the board, or else perform the algebra to separate the variables. Once the indefinite integrals appear, ask the students to take it from there in their pairs. Wrap up by pointing out the following salient features:

- Once all the *x*'s were on the right, and all the *y*'s were on the left, finding the relationship between *x* and *y* was just a matter of straight-forward antidifferentiation and algebra.

- The new idea here is that of rearranging the differential equation to get all the *y*'s on one side and all the *x*'s on the other side. This is called "Separating the Variables."

00:15-00:25 Activity: Combine the pairs into groups of four, and distribute the worksheet to them. (This worksheet asked the students to sketch the slope field for dy/dx = (xy)^(1/2), to use separation of variables to solve the differential equation symbolically, and then to use the formula that they had obtained to sketch several solution curves on the slope field.) Wrap up the activity by having two groups sketch their slope fields on the board, and having two other groups write their working for the separation of variables on the board.

00:25-00:30 Mini-lecture: When the students have finished, point out to them that two of the solution curves show behavior that is quite different from the other solution curves, in that dy/dx = 0 along these curves. Ask the class to describe these two curves verbally, and to find equations for them. When the students have done this, ask the class how they could have used the differential equation to identify these two "equilibrium solutions." Make sure that the following points are written on the board:

- The "equilibrium solutions" of a differential equation are the solution curves where dy/dx=0 at each point on the curve.

- You can find equations for the equilibrium solutions by solving the equation: dy/dx=0.

- Be careful what you cancel when you solve dy/dx=0 (otherwise you might miss some equilibrium solutions).

00:35-00:45 Activity: Have the students work on problem 5 from page 517 of their texts. (Because the slope field will be quite time-consuming to draw, encourage each person in the group to take one quadrant, and then share their results with the other members of their group.) While the students are working on this, draw some axes and sketch in the equilibrium solutions and slope field on the board. Encourage early

finishers to work on problem 14 from page 519 of the book. Wrap up the activity by having the students compare their slope fields and equilibrium solutions to the ones on the board and confirm that they were able to do this correctly.

00:45-00:50 Mini-lecture: Remind the students of some of the situations that they have applied differential equations to solve (build-up of drugs in a patient, growth of the human population, net worth of a company). Ask the class to think of some of these contexts and to imagine what the significance of the "equilibrium solutions" might be in these contexts. Have the students describe some of these to the class, and record the plausible answers in a table like:

Differential Equation models . . .	Equilibrium solutions represent . . .
Growth of a population	Carrying capacity of ecosystem
. . .	. . .

4.2 Example 2: LU Decompositions

Course: Linear Algebra. (Note: all references to the textbook refer to Lay [28]).

At this point in the semester, students had studied systems of linear equations and had become very proficient at setting up and solving systems with the assistance of their calculators. Students were familiar with matrix notation and matrix arithmetic. Students had used elementary matrices to describe row operations on coefficient matrices from systems of linear equations. Students had also completed a project in which they studied the computational efficiency of several algorithms by counting the number of floating point operations (flops) each algorithm used in various test problems. About one quarter of the students in the class had had much experience with writing mathematical proofs in other courses. Most of the students in the class had difficulty reading and understanding the more complicated proofs in their textbook, although all were able to perform calculations with concrete examples of matrices and vectors very competently. Students completed daily homework assignments, each of which included a proof-oriented question. In addition, class time had been devoted to discussions of what constituted a mathematical proof, and what was involved in constructing a proof.

Learning Objectives:

1. Students can articulate that solving upper or lower triangular systems of linear equations is a trivial matter of back substitution.

2. Students can explain why if it is possible to factor a given matrix A into the product of an upper and lower triangular matrix, then the system of linear equations $Ax = b$ can be solved easily without row reduction.

3. Students can row reduce a given matrix A to find an upper triangular matrix U that is row equivalent to A.

4. Students can use the row operations performed to construct U to construct a lower triangular matrix L such that $LU = A$.

5. From the algorithm, students are able to explain why it is that LU must equal A, and why L must be a lower triangular matrix.

Lesson Plan:

00:00-00:05 Activity: Put the following three systems of linear equations on the board and ask the students to solve them in the most computationally efficient manner that they can think of.

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -2 \\ -1.5 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 16 & 21 \\ 4 & 28 & 73 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$

Have the students report their results when they have had a chance to work the problems out. Ask the class how they went about solving each system of equations, and try to get them to be as explicit as possible on what "computationally efficient" meant to them. Make sure that students realize that although inputting the matrices into their calculators and row reducing is efficient in the sense that they don't have to do any mental calculations, it is not computationally efficient. If they were writing a computer program to solve these systems, row reduction is unnecessary, wastes flops, and can lead to round-off errors. Make sure that the students recognize that the first two systems can be solved by simple substitution with no need of row reduction whatsoever.

00:05-00:10 Activity: The students will have noticed some relationships between the numbers that they obtained when solving the linear equations. Tell the students that this is not just coincidence. Form the students into pairs and ask them to try to account for the relationships. (This will probably be quite hard for the students. One hint is to suggest that the students try multiplying the first two matrices together and see what they get.)

00:10-00:15 Mini-lecture: Use whatever the students have been able to come up with to explain the coincidences in their calculations. Use this explanation to show that once you know how to factor a matrix A into the product an upper (U) and a lower (L) triangular matrix, solving the system $Ax = b$ can be accomplished without row reduction by solving the two (easy) systems: $Ly = b$ and $Ux = y$. Tell the students that the rest of the lesson will be devoted to finding ways of calculating the matrices L and U.

00:15-00:25 Activity: Have the students row reduce the matrix:

$$A = \begin{bmatrix} 3 & -1 & 2 \\ -3 & -2 & 10 \\ 9 & -5 & 6 \end{bmatrix}$$

by hand (keeping track of the row operations that they use). When they have finished this, have the students work in pairs to construct elementary matrices that represent the row operations that they have used. Wrap this up by having the students report the upper triangular matrix that they have constructed and the elementary matrices that they have created.

00:25-00:30 Mini-lecture: Point out to the students how the upper triangular matrix, the elementary matrices and the matrix A are all related by an equation like:

$$E_n \ldots E_2 E_1 A = U.$$

Ask the students how they could make A the subject of the equation, and to carefully justify why the operations they suggest are valid (in particular, how they know that the elementary matrices are all invertible). Summarize the students' suggestions with an equation like:

$$A = E_1^{-1} \ldots E_n^{-1} U.$$

00:30-00:40 Activity: Have the students use the elementary matrices that they have found to calculate the product $E_1^{-1} \ldots E_n^{-1}$. Have the students call this matrix L and have them verify that $LU = A$. Wrap up by noting that not only do L and U multiply together to give A, but L is a lower triangular matrix as well.

00:40-00:45 Mini-lecture: Make a summary of the algorithm for finding U and L that the class has developed on the board.

00:45-00:50 Activity: Students will probably not have enough time to complete this activity during class. If this is the case, you could assign the balance of this activity for homework. Ask the students to work in pairs to see if they can use the description of the algorithm to come up with an explanation of why the matrix L calculated with this

algorithm must be a lower triangular matrix. If the students are really stuck, suggest some of the following hints:

- Each of the elementary matrices that you found appears to be lower triangular. Why is that?

- Let's say you have an invertible, lower triangular matrix. What sort of matrix is the inverse of that going to be? Try working the two by two and three by three cases.

- If you have two lower triangular matrices, what sort of matrix is their product going to be?

To wrap up, if students have made significant progress, tie everything together to explain how these three observations combine to guarantee that L will be a lower triangular matrix.

6. REACTIONS FROM INSTRUCTORS AND CONCLUSIONS

The lesson planning procedure described in this article and the descriptive analysis of the limiting practices of novice instructors that we have documented elsewhere [16] strongly suggest that current training methods, or lack thereof, are inadequate for moving instructors from their natural inclinations to teach as they have been taught (i.e. under the assumptions and limitations of a transmission model of teaching and learning [38]) to using SCI. A new component of training is needed to enable instructors to integrate the information on SCI that they receive in training programs in order to "learn to teach anew" rather than merely to "tack new teaching techniques on to ... existing lectures" [25, p. 112]. We believe that the lesson planning algorithm that we have described in this article provides a tool for achieving this.

We have introduced a number of novice instructors to this lesson planning algorithm via a training meeting. Our meetings have lasted for approximately one hour. Prior to the meeting, we asked the instructors to read a section from the course text that they planned to treat in the next week of the course. For the meeting, we created a seven page worksheet that described the algorithm to the instructors and left spaces for them to formulate learning objectives, outline activities, etc. Briefly, the goals of the meeting were to:

- Help instructors to learn to articulate "learning objectives" in an introductory college mathematics class.

- Help instructors learn to conceptualize lessons in terms of student learning objectives, rather than just by the material to be taught.

- Acquaint instructors with the lesson planning algorithm.

- Have the instructors plan an upcoming lesson using the lesson planning algorithm.

- Have instructor devise learning activities for their lessons that will encourage students to learn actively during the lesson.

We conclude with a list of ways in which the lesson planning practices of novice instructors using the lesson planning worksheet differ in highly significant ways from the kinds of limiting practices that we have summarized in Section 1. We also list some of the responses that we have collected from instructors who have been introduced to the lesson planning algorithm. The quotes are produced verbatim from interviews conducted with instructors who used the lesson planning algorithm to plan their classes. We have organized the quotations to follow the differences in practice that they highlight.

- Formulating a lesson in terms of student learning objectives promotes the inclusion of student learning activities as a natural and integral part of the lesson.

- This algorithm helps novice instructors to distinguish between the mathematical content and how students might learn the mathematical content. While we are not suggesting that these are unrelated, we note that novices tend not to consider the learners when planning lessons, in favor of worrying about subtleties of the nature of the material that may not even be suitable for presentation to students in introductory courses [3, p. 235].

- Creating a lesson based on learning objectives encourages instructors to develop an understanding of what it means for students to learn and understand mathematics when they create their lessons, rather than simply relying on thoughts of what it will take to present the mathematics in a way that the instructor finds satisfying.

> "The second goal of recognizing when the rule [should be used], versus the other rules, I think that really came out of my experience with watching the students, while observing the students who had just learned L'Hopital's rule, and then applied it to absolutely everything."

- In order to be able to decide on what learning activities to use to achieve specific students learning goals, instructors need to think about what the mechanisms or patterns of student learning are.

> "I think about the best ways to make the students understand the key concepts that will be introduced and also what would be really good examples to serve this purpose."

- The organization of a lesson around learning objectives encourages novice instructors to make student learning the focus of their thinking while they plan lessons, rather than their own performance.

> "I think even the one where I started out trying to do more of a lecture format wound up with, because these goals are much more about them being able to use the rules, that even in that lecture format I wound up doing much more like I lecture for five or ten minutes, now they would do problems."

- Having a well-described algorithm that instructors can complete to plan their lessons may encourage novice instructors to feel more confident in experimenting with the techniques of SCI of which they do not have personal experience.

> "...in terms of experience in the classroom, I've never experienced a math class like that [a student-centered mathematics class] so that's kind of treading on new ground ... on the plus side, part of the motivation of feeling that that was an okay direction to experiment with was having those materials [a version of the lesson planning algorithm made into a worksheet]."

- By formulating learning activities where students are actively working to learn the mathematics in the presence of the instructor, the instructor is able to collect accurate feedback on how the students are learning the mathematics, and if necessary, modify their approach to the material to help students with specific points of the material.

> "I think this one [the lesson planned with the lesson planning algorithm] is so much easier to assess ... because there's always a lot of, "Okay, now do it for this problem, now do it for this problem," and if they're getting it you can walk around and see that they've got it."

Finally, we point out that the use of the lesson planning algorithm provided here is intended to bring instructors to a point where the step-by-step use of the algorithm is no longer necessary for planning lessons. Rather, the kind of thinking involved in performing the algorithm is intended to internalize the identification and achievement of student learning objectives, while orienting the planning process around this central component. This was neatly summarized by one instructor who was introduced to the lesson planning algorithm as a novice,

> "…once I sort of understood the basic idea, I was able to do lesson plans on my own…. This is a basic schema which is good for beginners - once you have the basics of the method, you sort of transcend it, and use it implicitly without thinking about it. It's always in the background."

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