Homework 10

Remember that it is OK to work with your peers on the homework problems, but you should write up the solutions yourself. Also, if you do work with someone else on the homework, you should acknowledge that you worked with them on your write-up.

Problem numbers below refer to the course textbook, “An introduction to the theory of numbers”, by Niven, Zuckerman, and Montgomery.

As mentioned in class, this homework and the following ones will be less time-consuming than before, so that you can spend more time on your term paper.

1. Suppose that \( f(x, y) = ax^2 + bxy + cy^2 \) is a binary quadratic form. Prove that the discriminant
   \( \text{disc}(f) \equiv 0 \) or 1 modulo 4.

2. Conversely, suppose that \( d \) is an integer that is equivalent to 0 or 1 modulo 4. Prove that there is a binary quadratic form \( f(x, y) \) with \( \text{disc}(f) = d \). Hint: Consider the cases of \( d \equiv 0 \) (mod 4) and \( d \equiv 1 \) (mod 4) separately.