1. **Tosafot, Sukkah 8a**

   Whatever is one cubit square has a diagonal of 1 2/5 cubits. This figure is not exact, for it’s a little more. If you make a square of ten by ten, and divide it in two and again, you will find in it four squares of five by five. Now divide each of those squares with the diagonals that connect midpoints of the sides of the large square. You will find in the inside square an area of fifty square cubits, since it’s half of the outer one, since the 5 by 5 squares were split along their diagonals. But if the figure were only 1 2/5 cubits, it would be a 7 by 7 square, and this isn’t half the area of the outer square. It’s only 49 square cubits!

2. **Babylonian clay tablet YBC 7289** (ca. 1800–1600 BCE)

   From Wikipedia: The tablet depicts a square with its two diagonals. One side of the square is labeled with the sexagesimal number 30. The diagonal of the square is labeled with two sexagesimal numbers. The first of these two, 1;24,51,10 represents the number 305470/216000 ≈ 1.414213, an accurate numerical approximation of the square root of two. The second of the two numbers is 42;2 5,35 = 42 307/720 ≈ 42.426, the result of multiplying 30 by the given approximation to the square root of two.

3. **Mishnat Hamidot 4:1**

   השלוש מדות במשלשות ואלו הן: הנצבה החדה והפתוחה. איזו היא הנצבה? שני צדיה הקצורים מצורפים כל אחד בפני עצמו הם שוה והצד הארוך שהוא הקבע מצורף בפני עצמו. הצרוף האחרון לראשון.
There are three types of triangles: right, acute, and obtuse. What is a right triangle? Its two short sides are multiplied into themselves [and added together. The long side, which is the base, is multiplied into itself. The last square is equal to the first.

4. Mishnah Eiruvin 2:3,5

Mishnah 3. R. Judah said: [One may move objects on Shabbat in an enclosure that is] as large as two beit se’ah. [One beit se’ah = 50 cubits by 50 cubits, the area that can be planted with one se’ah of grain.] The Sages said to him: [the limit of] two beit se’ah was prescribed for a garden or a storage area only; but if [the enclosure] was a cattlepen, a sheepfold, a backyard, or a courtyard, it may be [as big as] five or ten beit kor. [One beit kor = 30 beit se’ah.]

Mishnah 5. R. Judah b. Baba said further: it is permitted to move objects in a garden or a storage area that is seventy cubits and a fraction by seventy cubits and a fraction and is surrounded by a wall ten handbreadths high, provided it contains a watchman’s hut or a dwelling place or it is adjacent to a town. R. Judah said: even if it contains only a cistern, a ditch, or a cave it is permitted to move objects within it. R. Akiba said: even if it contains none of these it is permitted to move objects within it, provided its area is no more than seventy cubits and a fraction by seventy cubits and a fraction. R. Eliezer said: if its length exceeds its breadth even by a single cubit it is not permitted to move any objects within it. R. Yose said: even if its length is twice its breadth it is permitted to move objects within it.

5. Babylonian Talmud, Eiruvin 23b

It seems that R. Akiva agrees with the first Tanna [i.e., his dimension statement is superfluous]! Actually, there is a small difference between them, as it is taught: R. Judah said: There’s a small difference [between two beit se’ah and a square of] 70 and a
fraction, but the Sages didn’t state it precisely. And how much is two beit se’ah? Like the courtyard of the Temple. How do we know this? R. Judah said: As it says in Scripture (Exodus 27:18), “The length of the courtyard was 100, and the width was 50 by 50.” The Torah is saying, take away 50 and surround them with 50. But what is the plain meaning? Abaye said: The Temple stood at the edge of 50, so that there were 50 cubits in front of it, and 20 in the remaining directions.

6. Jerusalem Talmud, Eiruvin 20b

R. Shmuel bar Nachman said in the name of R. Yonatan: From the dimensions of the Temple we learn: “The length of the courtyard was 100 cubits, and the width was 50 in/by 50.” And 50 times 100 is 5000. 70 by 70 is 5000 less 100, so we learn: 70 and a fraction. Shmuel taught: 70 2/3 cubits is two small. 70 times 2/3 and 70 times 2/3 are 140/3 and 140/3, which is 93 1/3. Coming out are 4/9 on the four corners. What remains is 19/3 less 1/9, as taught. This is a close approximation; the sages could not determine the exact dimensions of a [square] storage area whose area is two beit se’ah.

7. Rambam, Commentary on the Mishnah, Eiruvin 2:5

לפי שכבר נזכרו דברי ר' יהודה בן בבא הוסיף ואמר עוד אמר ר' יהודה בן בבא. וכבר ביארנו שחצר המשכן הוא בית סאתים. וכבר ידעת שתשבורת חצר המשכן היא בית סאתים חמשת אלפים אמה, לפי שארכו מאה ורחבו חמשים, וכל מקום שתשבורת שטחו חמשת אלפים אמה הרי הוא בית סאתים איך שתהיה תבניתו עגול או מרובע או משולש או שאר תבניות. אבל שטח מרובע בעל זוויות ישרות שתשבורתו חמשת אלפים אמה אי אפשר לדעת צלע אותו השטח אלא בקירוב, לפי וחמשת אלפים הוא מספר אין לו שורש, וששו בקירוב בשבעים אמה וחמש שביעיות אמה, והענין במספר הזה כמו שביארתי לך למעלה ביחס קוטר העיגול.Enqueueיפו, לפי גם הוא אי אפשר לעולם להגיע לידיעת שורש המספר "אלצם" שליו אלא בקירוב,ואי זה המنة והדרון, אלא שטענו את המספר כו, ופליקר אחרון שתריע את השיר, ולפי ושעה שביחד עם ש сохרים על שבעים וחמש שביעיות, לפי שאם תเทศ את השירים חמישת שביעיות כמו שביארתי לך ותקפוץ שבעים וחמש שביעיות בשבעים וחמש שביעיות יהיו תוצאות המספר חמשת אלפים וחצי אחד בקירוב. ולא אם תเทศ השירים שני שלישים כמו שנראה מן הירושלמי יהיה מדת אותו השטח ארבעת אלפים ותשע מאות ותשעים ושלשה ושבע תשיעיות, וזה הוא ההבדל שבין דברי ר' יהודה בן בבא שאמר בעיתים ו畬ים על שבעים ו畬ים, ובין דברי ר' עקיבא שאמר בלבד שתהא שבעים ו畬ים על שבעים ו畬ים נוסף על מחלוקת שביניהם בשומירה ובית דירה, שרי' יהודה בן בבא מדקדק מאד בחשבון עד שיהא מדת המרובע בית סאתים שלמים לפיכך היה עושה השירים חמישת שביעיות כמו שאמרנו או משהו יותר מדוייק מזה. ור' עקיבא היה עושה החשבון בקירוב יותר, והעשה השירים שני שלישים או משהו קרוב לזה עד שיהא מדת המרובע קרוב לבית סאתים, וכך ייראה מדבריהם במקום אחר.

לפי שאמר ר' יהודה אפילו אין בה אלא בור ושיח ומערה.
We already explained that the Temple courtyard is two beit se’ah. And you know that the area of the Temple courtyard is 5000 square cubits, since its length is 100 and its width is 50, and any place whose area is 5000 square cubits is two beit se’ah, whether it’s a circle or a square or a triangle or any other shape. But it is not possible to know the side a square with right angles whose area is 5000 square cubits exactly, since 5000 is a number that doesn’t have a [rational] square root. Its root is approximately 70 5/7, and the significance of this number is like I explained above about the relation of the diameter of a circle to its circumference ... and this isn’t because of a lack of our knowledge, but the nature of this number. And therefore they said “70 and a fraction” (shirayim). If you make the fraction 5/7 like I said, and multiply 70 5/7 by 50 5/7, you will get approximately 5000 ½. But if you make the fraction 2/3 like in the Yerushalmi, the area will be 4993 7/9. This is the difference between R. Yehudah ben Bava, who said “70 and a fraction by 70 and a fraction,” and R. Akiva, who said “only 70 and a fraction by 70 and a fraction,” in addition to the presence of a hut or a booth. R. Yehuda ben Bava is very precise in computation, so that it should be two full beit se’ah, so he makes the fraction 5/7 like we said, or something more precise than that. R. Akiva gives a more approximate figure, and makes the fraction 2/3 or something close to that, so that the area is close to two beit se’ah, and thus is seen in their words elsewhere.

8. R. Saadia Gaon (882-942), Kitāb al mawārīth (Laws of inheritance, trans. Solomon Gandz)

[Background: Finding the diameter of a circle whose area is 9 kab = 1.5 se’ah = 3500 square cubits. The claim is that \( \sqrt{5000} \approx 70 \frac{41}{60} \).]

If it is circular, we must add one-third to the measure of these 9 Kab, and therefore, also add one-third to the lot; thus the area which was originally 3750 becomes 5000. So that if you seek the measure of (the diameter of) the circular field, you will multiply 70 + 2/3 + ½ of 1/20 by itself and find the true sum. Its explanation is as follows: 7 by 7 is 49; 7 by 70 is 490, 70 by 70 is 4900. There still remains 2/3 + ½ of 1/10: 2/3 of 70 is 46 2/3; ½ of 1/10 of 70 is 3 ½; together they are 50 1/6; double it, for it is multiplied by itself, and you get 100 1/3. Throw away the one-third to the other fractions and 100 remains; add it to the above and you get 5000 conforming to the computation. Such is the procedure for the circular field.

9. Deuteronomy 22:9

לֹא־תִּזְרַע כַּרְמְךָ כִּּכָּלָיִם פֶּן־תִּקְדַש הַמְלֵאָה הַזֶּרַע אֲשֶּֽר־תִּזְרָע וּתְבוּאַת הַכָּרֶם׃

You shall not sow your vineyard with a second kind of seed, else the crop—from the seed you have sown—and the yield of the vineyard may not be used.

10. Mishnah Kilayim 5:5
If one plants or maintains vegetables in a vineyard, this prohibits 45 vines. When does this apply? When they are planted 4 by 4 or 5 by 5. If they were planted 6 by 6 or 7 by 7, it forbids 16 cubits in all directions, in a circle rather than a square.


4 by 4 or 5 by 5 – If one vine to another is 4 or 5 cubits, then 45 vines are forbidden. For if one plants 7 rows of vines, with 7 vines in each row, spaced 4 cubits apart, you will have 49 vines in a square 24 by 24 cubits, since there are 6 gaps of 4 between each row. And if one plants [vegetables] around the middle vine of the fourth row — i.e., the middle vine of all the vines — it forbids 16 cubits in all directions, as taught in the second half, i.e. a circle 32 cubits in diameter. All 49 vines are inside this circle except the four corner ones, for “a square of a cubit is a cubit and two fifths in diagonal.” We find that a square of 24 has a diagonal that is 9.6 cubits greater [i.e. 33.6 cubits], so the corners are outside the 32.
you tie a thread on the corner and bring it on the diagonal [of a 20x20 square], and then straight down 10 cubits, we find the length of the thread is 38 cubits, for the diagonal of 20x20 exceeds the width by 8. And if you stretch it diagonally from one corner to the other, it is totally clear that it shortens greatly. . . . And from our Mishna, we can conclude that the diagonal of 30x20 does not exceed the width by more than 2 cubits [i.e. isn’t greater than 32], for the distance [from the vine next to the corner] to the middle vine is 16 cubits.

People who are wise in geometry have said that the sum of the squares of the two sides equals the square of the diagonal. That is, you measure the width and make a square of that size, and you measure the length and make a square of that size, and you measure the diagonal and make a square of that size. Then the area of the diagonal equals the area of the other two squares. To see this, make a square of 100x100, and divide it into four squares of 50x50, and then divide each of them diagonally. Your eyes will see that the inner square has half the area [of the outer square], for we’ve divided each of the 50x50 squares in half. Thus, the squares of the two sides are twice 50x50. And this also the square of the diagonal, as I showed you, since the inner square is half of the outer one, or 100x50, which is twice 50x50.
something that doesn’t add up exactly: the statement that a one cubit square is one and two fifths cubits in diagonal. As you can see, with these two squares, one inside the other, using the figure of one and two fifths, the inner one is $70 \times 70$. Its diagonal is 100 since it’s the width of the outer square, but it’s only 98 by computation. The sages were not concerned with such small amounts, so both the words of the sages and the words of the geometers are valid.

And even though I’ve proven this for a square whose sides are equal, it can’t be proved for a rectangle whose length is greater than its width, and therefore, there is nothing to it. For the diagonal of 30 by 20 is less than 32, as our Mishnah shows. And if you make a square of 32 by 32, it’s too small; it’s not the size of two squares, one 30 by 30 and one 20 by 20.

12. Tosafot, Bava Batra 102a

The diagonal of a 4x6 rectangle is at most 2 cubits [larger than the length], close to the diagonal of a 5x5 square. For in a 4 by 6 rectangle, stretch the thread on the diagonal of a 4x4 square, which is 5 3/5 cubits. There are still 2 cubits left below, so in all it’s 7 3/5 cubits. All the more so, if the thread goes straight along the diagonal of the 4x6 rectangle, it will be less than this. And the diagonal of a 5x5 square is even greater, since the area of the 5x5 square is 25 while the area of the 4x6 rectangle is 24.

If there are 5 cubits from vine to vine, it sanctifies 45 vines. Rabbeinu Shimshon explained that from the middle vine to the vine next to the corner is no more than 16 cubits, and therefore it forbids 45, just as if they were spaced 4 cubits apart. But this is obviously incorrect. From the middle vine to the vine on the side is 15 cubits, and half the diagonal of a 30 by 30 square is 21 cubits. If from the middle vine to the vine next to the corner is only 16 cubits, when you stretch a thread of 21 cubits from the middle vine to the vine next to the corner, and then stretch the thread from this vine, you arrive at the vine in the corner with remaining thread, since they’re only 5 cubits apart. Thus it’s more than 16 cubits between them. According to the theorem of the geometers that the sum of the squares of the sides equals the square of the diagonal, which is true even for a rectangle whose length is greater than its width as I will illustrate for you [in a figure], from the middle vine to the one next to the corner vine is 18 cubits.

Rambam explained that it ruins the vines next to the corner vines even though they are more than 16 cubits away from the middle vine, because we view the 16 cubits around the middle vine as if they were filled with vegetables, and this circle is 32 in diameter and sticks out of the 7 rows of 5 cubits by one cubit on each side, since there are 6 gaps between 7 rows, each 5 wide, and thus it’s a 30 by 30 square. The diagonal is 32 cubits, so the circle sticks out of the 7 rows by one cubit, and thus comes up to the tending region of the eighth and ninth rows. Therefore, we must add to the circle 4 cubits in every direction, so it’s a circle of 40, since we view the circle of 32 as if it were filled with vegetables everywhere. Wherever it enters the tending region, it ruins. Therefore, we forbid the 8 vines next to the corners, even though they are more than 16 cubits from the middle vine.


If one sows a vegetable or grain in a vineyard, or allows it to grow, it forbids the vines around it within 16 cubits in all directions, in a circle rather than a square. And we
view this circle (whose diameter is 32 cubits) as if it were completely full of vegetables. And any vine that lies within this circle is forbidden, and any vine that is outside the circle is not forbidden.

When does this apply? When the distance from the edge of the circle to the next row of vines outside it is more than four cubits. But if the distance between them is four cubits or less, we view the circle as if it stretched to the next row and is if its diameter were 40 cubits, and any vine that falls within this circle becomes forbidden.

15. **Kesef Mishneh, Hilchot Kilayim 6:2** (R. Joseph Caro, 1488-1575)

I have found a letter in which Rosh asked R. Yisrael b. Yosef, who was very knowledgable (chacham b’chochmot), who is correct in this matter (Rambam or R. Shimshon). R. Yisrael writes: “My lord, my master, may your prestige be magnified like your talent and ability. I consulted Rambam’s (z”l) commentary on the Mishnah, and I saw that that is knowledge is extraordinary. His explanation provides excellent guidance, but it seems, because of my lack of knowledge, that he elaborates too much and the language is not clear and would be confusing to someone who doesn’t study it carefully. But it all makes perfect sense. One may not stray left or right from his approach. Thus, I will explain the entire argument and not add anything, save for simplifying the language…

“This is what I have understood from the Rambam’s (z”l) explanation of this Mishna, and it isTorah l’Moshe b’Sinai. But let not my lord be seduced by R. Shimshon’s (z”l) explanation of this Mishnah, and do not pay attention to it at all, for it is beyond his honor, a disgrace founded on a vacuous line [of thought] and a nonsensical foundation. What the geometers have written is correct. His claim that [the Pythagorean theorem] cannot be proven for a rectangle whose length is greater than its width is incorrect — it can! His words have no merit. May your peace be like a river and endure for your offspring, according to your will and the wish of your young student touched by your mercy. Yisrael bar Yosef.”
And we do not know what to do.” To which of the holy scholars shall we turn in understanding this Mishnah? For among all the writings of the commentators, may they be granted eternal life in the World to Come, there is nothing which we can actually rely on. The words of Rabbeinu Shimshon are utter nonsense, and the words of Rambam do not fully explain the Mishnah, as we have shown. And it is clear from his essay that one has to keep changing the language and the phrasing to account for every single objection. Likewise, the words of the Raavad don’t make any sense and don’t explain the Mishnah, and the author of the Kesef Mishnah didn’t get a single thing right.

Rather, there is a clear explanation for why 45 vines are forbidden in the 5x5 case: it is a stringency imposed by the Sages because an ordinary vineyard is planted 4x4. If one were to see a vineyard planted 5x5, with a vegetable planted in the middle that doesn’t forbid 45 vines but rather 37, one might not be aware that it wasn’t an ordinary vineyard, since the difference between 4x4 and 5x5 is small. One might conclude that it was an ordinary vineyard, and that in an ordinary vineyard a vegetable only forbids 37 vines. Therefore, the sages said that in a 5x5 vineyard, the rule should be the same as in an ordinary 4x4 vineyard. But in a 6x6 vineyard, for which there’s a big difference (the spacing between rows is 50% bigger than in an ordinary vineyard), there is no risk of confusion, since everyone knows that this isn’t an ordinary vineyard, and there is no need to impose an extra stringency. And all of the complicated cases considered by Rambam and Raavad are not necessary. And we have a simple explanation for why the Mishnah is given in two separate statements, specifying 45 vines in the first and 16 cubits in the second: the rule in the 5x5 case is made more stringent because of the 4x4 case.
17. **Netzach Yisrael** (R. Israel ben Moses Segal of Zamość, c. 1700–1772)

We have stated that the count in the 4x4 case is 45 and no more. But now let us assume that the vegetable is planted in the space between the vines. We find that there are 52 vines inside the circle! For there are eight rows of vines which enter the circle, with seven gaps of 4 cubits, making 28 cubits, and the circle protrudes by 2 cubits on all sides. And all the vines in these 8 rows fall within the circle, except for the three on each corner, for a total of 52 vines… And since the Mishnah says that only 45 vines are prohibited, even in the 4x4 case, we are forced to conclude that the Mishnah specifically assumes that the vegetable is planted at the central vine. This is also clear from the language of the Yerushalmi, which says that the vegetable is planted at the central vine, and from the Tosafot Yom Tov…

This must also hold in the the 6x6 case. We have explained that the Mishnah specifically deals with the case that the vegetable is planted around the middle vine. If so, there are only 21 vines, not 24 as Rambam thought! … And this is just like in the 7x7 case, where the number of prohibited vines is also 21. Why, then, does the Mishnah include the count of 45 vines in the 4x4 and 5x5 cases, but not in the 6x6 and 7x7 cases, in both of which it’s 21 vines? Why did it just state the 16-cubit rule in the 6x6 and 7x7 cases, if the counts are the same for both? This is a serious challenge for all the commentaries and all the opinions, but especially for Rambam, who stated that the count in the 6x6 case is 24 (since he assumed that the vegetable is in the middle space) but that it’s only 45 in the 4x4 case (which requires the vegetable to be at the middle vine).

And now, all those who are interested in these matters, give ear to my words, and you will find comfort for your soul in the resolution of the confusion raised above. In the first chapter of Tractate Eiruvin, it says: “Abaye said in the name of Rav Nachman: The cubit of the sukkah and the cubit of the alleyway are a cubit of 5 [handbreadths], and the cubit of kilayim is a cubit of 6 [handbreadths].” And Rashi wrote about this: “Since it says this is big and this is small, we deduce this is strict and this is strict.” That is, what Abaye is saying is not really that the laws of the sukkah and the alleyway require a cubit of 5 and Kilayim uses a cubit of 6, but rather that in both settings, whenever using a 5-handbreadth cubit produces a stricter outcome, we use a cubit of 5, and whenever using a cubit of 6 produces a stricter outcome, we use a cubit of 6…

And now we can finally explain the meaning of our Mishnah. The cubits used for stating the distance from the vegetable are cubits of 6 handbreadths, since that produces the stricter outcome, while the cubits used for stating the distances between the rows are cubits of five handbreadths. And now we can truly understand the Mishnah’s ruling that in the 5x5 case, the vegetable prohibits 45 vines. The distance from the central vine to the vine by the corner is now the diagonal of 10 by 15 cubits, which is just about 18 cubits of 5 [Note: \(\sqrt{10^2 + 15^2} = 18.027 \ldots\)], which is 90 handbreadths, which is 15 cubits of 6! Thus, they fall within the circle and are prohibited. But the four corners vines are at a distance of the diagonal of 15 by 15, which is about 21 [\(\sqrt{15^2 + 15^2} = 21.213 \ldots\)] which must be outside the circle [since it’s about 17.7 cubits of 6].

But we must now ask: If we assume that the distance from the vegetable is measured in cubits of 6, and the distances between the rows are measured in cubits of 5, what would happen in the 4x4 case? We would end up with more than 45 vines
prohibited! For the four vines in the corners are now only about 17 cubits 
[\sqrt{12^2 + 12^2} = 16.97 \ldots] from the center in cubits of 5, and in cubits of 6 they are inside the circle [since it’s about 14.1 cubits of 6].

The answer comes from analyzing the principle of ruling toward stringency. We sometimes measure with cubits of 5 and sometimes with cubits of 6 because we are unsure of which to use. But we know [from earlier in Kilayim] that a vineyard planted with spacing of less than 4x4 is not a vineyard, according to Rav Shimon. The meaning of “not a vineyard” is that it’s permissible to sow it with seeds… Therefore, in the vineyard planted 4x4, the vegetable cannot prohibit more than 45 vines by process of elimination [ממה נפשך]. If the sages were working in cubits of 5 [for the radius], the vegetable would only forbid 16 cubits around it, in cubits of 5. And if they were working in cubits of 6, the vineyard would be planted in rows of less than 4 cubits, and therefore the vegetable would not forbid anything at all!

And now we can resolve the final issue: When the vineyard is planted 6x6, it no longer forbids 21 vines, just as in the 7x7 case. In the 6x6 case, the four vines in the corners are only about 17 cubits [of 5] away, and when we convert to cubits of 6, they enter into the circle of 16 cubits. But in the 7x7 case, the four vines in the corners are about 20 cubits [of 5] away, and even when we convert to cubits of 6, they are still outside the circle. We thus deduce that in the 6x6 cases, 25 vines are forbidden, and in the 7x7 case, 21 are forbidden. And we have now correctly interpreted the Mishnah.