Jewish Geometry
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Part I: The Diameter, Circumference, and Area of a Circle


The reason behind all of these computations – why we add this number or subtract that one, and how each fact is known, and the proof of each fact – this is the science of astronomy and mathematics (gematria), about which the Greek scholars wrote many books that are available to scholars today, while the books written by the scholars of Israel in the days of the Prophets, the children of Issachar, have not survived for us. But since all of these facts have clear and flawless proofs, which a person cannot possibly doubt, we do not care whether they were written down by the Prophets or the gentiles. If a statement comes from clear reasoning and a flawless proof, we do not concern ourselves with who said it but only with the reasoning and the proof.

2. I Kings 7:23-26

Then he made the molten sea, ten cubits from brim to brim, completely round. Its height was 5 cubits, and a line of 30 cubits went around it. There were gourds below the brim, completely encircling it—ten to the cubit, encircling the sea; the gourds were in two rows, cast in one piece with it. It stood upon twelve oxen: three facing north, three facing west, three facing south, and three facing east, with the tank resting upon them; their haunches were all turned inward. Its thickness was a handbreadth, and its brim was made like the brim of the cup of a lily; it contained 2000 bat.
3. **Commentary of R. Levi b. Gershon (1288–1344), I Kings 7:23**

This is an approximation, for the circumference of a circle is greater than three times the circumference by approximately a seventh of the diameter. And if we say that the measurement of the circumference was taken on the inside of the basin, this is closer to the truth, but still just a close approximation. The thickness is a handbreadth, so the inner diameter is ten cubits less a third, and the inner circumference is closer to 31 1/3 cubits.

4. **Mishnat haMidot 5:3–4 (Rabbinic text on geometry, ca. 150 CE?)**

What is the flat one [i.e., the circle]? That is one that is put down [flatly] on the ground like a circular field or a circular figure. If one wants to measure [the area], let him multiply the thread [diameter] into itself and throw away from it the one seventh and the half of a seventh; the rest is the area, its roof. And if you want to know the circumference all around, multiply the thread into 3 and one seventh and it amounts to 22. And if you want to compute the area, take half of the circumference, which is 11, and multiply it into half the thread which is 3 and a half, and it amounts to 38 and a half. It is the same [result] according to the first [method] and the last one.

Now it is written: “And he made the molten sea of ten cubits from brim to brim, round in compass,” and [nonetheless] its [circumference] is thirty cubits, for it is written: “And a line of thirty cubits compassed it round about.” What is the meaning of the verse “and a line of thirty cubits…”? Since the people of the world say that the circumference of a circle contains three times and a seventh of the tread, take off from that one seventh for the thickness of the sea on the two brims, then there remain “thirty cubits compass it round about.” The seas, reservoirs, and cistern are equally [computed] according to this measure in length, breadth, and depth. Thus you have learned the measure of the circular [figure].
5. **Mishnah Eiruvin 1:4–5**

The cross-beam of which they [the rabbis] spoke must be wide enough to hold an ariach, which is half of a brick three handbreadths [wide]. It is sufficient for a beam to be one handbreadth wide in order to hold the width of an ariach. [It must be] wide enough to hold an ariach and strong enough to support such an ariach. R. Judah ruled: [It is valid if it is] wide enough but not strong enough. If it was made of straw or reeds, we view it as though it were made of metal; [if it was] curved we view it as though it were straight; [if it was] round we view it as though it were square. *Whatever has a circumference of three handbreadths has a diameter of one handbreadth.*

6. **Babylonian Talmud, Eiruvin 14a**

"If it is round we view it as if it were square." Why do I need this? The next part is necessary for it: "*Whatever has a circumference of three handbreadths is one handbreadth in diameter.*" Whence these words? R. Johanan replied: Scripture stated: “And he made the molten sea, ten cubits from brim to brim, completely round. Its height was five cubits, and a line of thirty cubits went around it.” (II Kings 7:23) But what about its brim? R. Papa replied: Its brim was [as thin] as the flower of a lily, as it is written: "It was a handbreadth thick, and its brim thereof was wrought like the brim of a cup, like the flower of a lily; it held two thousand bat." (7:26) But isn’t there a little? — When it was computed, it was that of the inner circumference.

7. **Tosafot haRosh, Eiruvin 14a** (R. Asher ben Yechiel, Germany/Spain, 1250–1327)

"When it was computed, it was that of the inner circumference."
“Whatever has a circumference of three has a diameter of one handbreadth. Whence these words?” I am baffled why it is necessary to ask this question regarding something that is clear to the eyes. A person can get an object that is one handbreadth wide and measure its circumference! It seems to me that it is meant to explain that the measurement is not exact, since the circumference is greater than three handbreadths. So the Talmud asks “Whence these words?” to permit the Sages to give a bound even though it’s not exact, but it’s close to being exact. They bring a proof from Scripture that the Solomon’s sea was the same: it was ten cubits in diameter, yet Scripture says that a line of thirty cubits encircled it, even though it was actually longer. And the Talmud objects, “What about its width?” For when Scripture says that a line of thirty cubits encircled it, it includes the brim, and therefore the line was longer than thirty cubits. And how can the verse say something that’s so contestable? And it further asks “But surely there was something?” Even though it was like a lily, they still object that it was greater than thirty cubits.

8. Rashi to I Kings 7:26

2000 bat. This is 6000 se’ah, for the bat is three se’ah, as it says: “The bat and the ephah shall be the same measure” (Ezekiel 45:11). We find 150 mikva’ot: 4000 se’ah are 100 mikva’ot, and 2000 se’ah are 50 mikva’ot. And when you divide it all up according to the Sages’ computations (namely 1x1x3 cubits for each mikva), you find it is thus: the lower 3 cubits were 10 cubits square, hence 100 mikva’ot, and the upper 2 cubits were round, hence 50 mikva’ot, since the area of a square is bigger than the area of a circle by a quarter. And in II Chronicles 4:5, it says: “It held 3000 bat.” And our Sages explained that it held 3000 bat of dry measure, since the heap is one-third the capacity of the vessel.

9. Babylonian Talmud, Sukkah 7b–8b

R. Johanan said: If a sukkah was [round] like a furnace, if twenty-four men can sit around its circumference it is valid, and otherwise it is invalid. According to whom is this view? Obviously according to Rabbi, who says that a sukkah which is not four cubits square is invalid.

But consider: A man occupies the space of a cubit, and whatever has a circumference of three handbreadths has a diameter of one handbreadth. Twelve should be enough! — That applies in the case of a circle, but in the case of a square, a greater perimeter is required.

But consider: By how much is a square greater than a circle? By a quarter. Sixteen should be enough! — That applies in the case of a circle inscribed within a square, but a square inscribed within a circle a greater circumference requires more on account of the projection of the corners.

But consider: If the side of a square is a cubit, its diagonal is approximately one and two fifths cubits. Sixteen and four fifths should be enough! — [R. Johanan] gave only an approximate figure.

We could say that [R. Johanan] gave an approximate figure when the discrepancy is small, but could we make such an assumption when the discrepancy is big? — Mar Kashisha the son of R. Hisda said to R. Ashi: Do you think that a man occupies one cubit? [The fact is that] three men occupy two cubits. How much is it [the circumference of the circle]? Sixteen cubits; and we demand here sixteen and four fifths, but [R. Johanan] gave only an approximate figure.

We could say that [R. Johanan] gave an approximate figure only for a stringency, but could we make such an assumption for a leniency? — R. Assi answered R. Ashi: In truth, a man occupies a cubit-space, but R. Johanan does not include the space occupied by the men. How much is this? Eighteen; and sixteen and four-fifths suffice. Thus he only gave an approximate figure, in the direction of stringency.

The rabbis of Caesarea (some say: the judges of Caesarea) say: A circle inside a square is [less by] quarter, and a square inside [that] circle is [less by] half. But that’s not relevant, since it can’t be that much.

10. Tashbetz I.165 (R. Shimon ben Tsemah, Spain/Algeria, 1361–1444)
You brought a proof from the language of the Talmud, which says: “He was imprecise for a stringency.” They didn’t say this because of this geometric precision, but rather even in their essence they were stringent that it wasn’t precise. As it says in the first chapter of Sukkah (7b) about a sukkah in the shape of a furnace. That discussion depends on two principles: Whatever has a circumference of three handbreadths has a diameter of only 7 2/3 cubits, and the diagonal of a four cubit square is 5 2/3 cubits, roughly, and when you take away from this two cubits because of the people, we’re left with 5 2/3 cubits, and the diagonal of a four-cubit square is 5 3/5 cubits, roughly. And the words of R. Johanan come to be very close to the knowledge of the geometers, for 3/5 is roughly, and when you take away from this two cubits for the people on each side, six cubits are left, and the people are outside, then the diameter is eight cubits since its circumference is 24. And when you take away two cubits for the person on each side, six cubits are left, and the people are outside, then the diameter is eight cubits since its circumference is 24. For a sukkah with a circumference of 24 cubits has a diameter of only 7 2/3 cubits, and the diagonal of a four cubit square is 5 2/3 cubits, roughly. And when you take away two cubits for the person on each side, six cubits are left, and the people are outside, then the diameter is eight cubits since its circumference is 24. And when you take away two cubits for the person on each side, six cubits are left, and the people are outside, then the diameter is eight cubits since its circumference is 24.

And if the sages were worried about these geometric precisions, as you say, and they were trying to correct the statement of R. Johanan to make it close to the truth, they should have mentioned this here and said that even though, according to the principles that we laid out, there are in R. Johanan’s words an extra two fifths, yet in truth there is no noticeable excess. For in a square of one cubit, the diagonal is more than a cubit and a square of one handbreadth in diameter has a circumference of three handbreadths, and a square of one cubit in diameter has a circumference of three handbreadths. And in this, the words of R. Johanan are amazingly close to the truth.

That’s the summary of their words, even though they didn’t say it in that language.
We could say one of two things: One, this could just be the Sages’ received tradition, to work in this manner. And even though it’s an approximation, these measures are halacha given to Moses at Sinai. And we could say that this is how halacha given to Moses at Sinai is stated. The Torah was not given to the ministering angels! And perhaps the halacha transmitted to them was that they should work with these principles, even though they’re approximate, as if they were precise. And there is support from this from the sea of Solomon, which Scripture describes with an approximation, as I’ve explained. This is one explanation. Or one could say that they used these values to instruct students, as it says: “One should always teach one’s student in the easiest way” (Pesachim 3b, 63b). But for actual use, we should use the precise, true figure, which was passed to the Sages by those who knew the measures. It seems the halacha is taught to beginning students and the actual use is taught to wise ones in order to be precise about the true value. And I think this is the correct way to interpret the Sages’ words.

11. Rambam, Commentary on the Mishnah, Eiruvin 1:5

You need to know that the ratio of the circle’s diameter to its circumference is not known, and it is never possible to express it precisely. This is not due to a lack in our knowledge, as the fools think, but it is in its nature that it is unknown, and there is no way to know it. But it can be measured approximately, and the geometers have already written essays about this, that is, to know the ratio of the diameter to the circumference approximately, and the proofs for this. The approximation which is accepted by the educated people is the ratio of one to three and one seventh. Every circle whose diameter is one handbreadth, has in its circumference three and one seventh handbreadths approximately. As it will never be perceived but approximately, they took the nearest integer and said that every circle whose circumference is three fists is one fist wide, and they contented themselves with this for their needs in the religious law.
“By how much is [the area of] a square bigger than [the area of] a circle? By a quarter.”

We cannot prove this principle from the fact that a square of 3 by 3 has a circumference of 12, and a circle of diameter 3 has a circumference of 9 (since whatever has a circumference of three has a diameter of one, as stated in our Gemara). That is, we can’t just prove [the area formula] from the fact that the perimeter [of a square] is bigger by a quarter [than the circumference of the inscribed circle]. Do you really think that a round disk of diameter 4 has the same area as a 3x3 square just because their perimeters are the same? If you divide a 3 by 3 square into 3 strips lengthwise, and 3 widthwise, you’ll find that it’s only 9 square cubits, whereas a round disk of diameter 4 has an area of 12! (For if a square of 4 by 4 is divided into 4 pieces lengthwise and widthwise, it will have 16 square cubits. And a square is bigger than a circle by a quarter, so you see that the circle has an area of 12.) There is really no proof from the circumference at all!

Likewise, a strip that is 5 by 1 has a perimeter of 12, and when you divide it into 1 by 1 pieces, its area is only 5. And this is the reason: when you put a thread on a square, it narrows out at the corners, and when you put it on a circle, it spreads out.

There is no proof from the circle’s circumference at all. We cannot prove the area formula of a square just by the circumference of the circle.

By how much is a square bigger than a circle? By a quarter.
And if we wish to determine the exact amount by which a square is bigger than a circle, we will be able to prove it as follows. Take a point, and encircle it with many threads, one around another until you fill up a circle of 1x1. And then cut the threads downward, along the radius of the circle, and spread out the threads to the left and to the right. We find that each thread is longer than the previous thread on both sides, all the way to the topmost thread, which has length 3 (since it is the circumference of a circle of diameter 1). We find the threads arranged as follows, like a [triangular] strip whose width is ½ and which tapers to a point in either direction.

If you now divide this in the middle (near the original center point), you will find two [triangular] strips, each of which is 1½ long, and its width is ½ at one end and tapers to a point at the other end. And now join these two together, putting the long side of one next to the short side of the other. You will find a [rectangular] strip with length 1½ and width ½. Divide it into three equal pieces, each of which is a square, ½ by ½. And if you divide a 1x1 square in quarters, you will find four square regions of ½ by ½. Thus, the area of the square exceeds the area of the circle by a quarter.


We must now ask: Why is the sea described in Kings in such a way that seems to the reader to be so imprecise? Must one who reads the verse there think that the circumference of the sea was only thirty cubits and not realize that the prophet knew that its circumference was more than 31 cubits in reality, and that the figure thirty cubits was given only for halacha? Is there any sign in the verse of our precision? Let us look carefully at the above-mentioned verse, on which the figure of 3:1 is based. After it says “perfectly round,” it mentions the height of the sea, and then adds: “And a line of thirty cubits encircled it all around.” One may ask: Why doesn’t it say, more briefly, “It was round, thirty cubits around” instead of the long version: “And a line of thirty cubits encircled it all around?” It seems to me that the author chose this long phrase because he wanted to add the word וְ, which has both a pronounced (k’ri) and a written (k’tiv) form. The written form is וְכָל, and the pronounced form is כָל. What can we learn from the pronounced and written forms? When we are speaking about numbers, we explain that the difference between the versions can be found in a numerical manner, in *gematria*. The *gematria* of כָל (the conjunctive vav isn’t part of the word) is 111. The *gematria* of וְ is 106. A mathematical calculation of the circumference of the sea, based on a diameter of 10 cubits, equals $10 \times \pi = 31.4159265\ldots$. If we use the estimate 3330/106, we obtain a number that is closer to the correct number than the number obtained using 220/7 as usual. The ratio 330/106 : 30 is equal to the ratio 111 : 106, the *gematria* values as above of כָל and וְ. The closeness is so surprising as to amaze.

If we compare 31.4159265 : 30 = 1.0471975 to 111:106 = 1.0471698, we see that the difference is smaller than three parts in 100,000, an extremely good estimate.

What did the prophet want to teach us with these written and pronounced forms? The written form serves as a sign of the exact value of the circumference, and the pronounced form teaches us what we need to know for halacha. The written form is what we see with our physical eyes, and therefore it is connected to the number that we find if
we measure the circumference. The pronounced value is what gives us knowledge for matters of halacha. If we explain the written form in this manner, we cannot say that the prophet did not give us the exact value of [the circumference of] the sea properly in his statement that “a line of thirty cubits encircled it all around.” Really, he is telling us the circumference in accordance with our reality, albeit only with a hint; his main intention was to teach us halacha.

One may ask: Why isn’t the word is וְקו written with the same written and pronounced forms in the other verse as well (II Chronicles 4:2)? We would answer, that there is no need to repeat the sign of what is in accordance with our reality. And one may ask: Don’t we also find written and pronounced forms of וְקו in two other places in the Bible: “And the line will go straight out” (Jeremiah 31:38), and “The line is being applied to Jerusalem” (Zechariah 1:16)? We answer that we do not have to explain all written and pronounced forms in the same way.


That circle—which, begotten so, appeared in You as light reflected—when my eyes had watched it with attention for some time, within itself and colored like itself to me seemed painted with our effigy, so that my sight was set on it completely. As the geometer intently seeks to square the circle, but he cannot reach, through thought on thought, the principle he needs, so I searched that strange sight: I wished to see the way in which our human effigy suited the circle and found place in it—and my own wings were far too weak for that. But then my mind was struck by light that flashed And, with this light, received what it had asked. Here force failed my high fantasy; but my Desire and will were moved already—like a wheel revolving uniformly—by the Love that moves the sun and the other stars.