Question 1. Find conditions on $a, b, c$ so that $v = (a, b, c)$ in $\mathbb{R}^3$ belongs to $W = \text{span}(u_1, u_2, u_3)$, where

$$u_1 = (1, 2, 0), \quad u_2 = (-1, 1, 2), \quad u_3 = (3, 0, -4)$$
Question 2. Find the general solution of the system

\[
\begin{align*}
    x_1' &= 2x_2 + 2x_3 \\
    x_2' &= 2x_1 + 2x_3 \\
    x_3' &= 2x_1 + 2x_2
\end{align*}
\]
Question 3. Find a basis and dimension of the subspace $W$ of $\mathbb{R}^3$ where
(a) $W = \{(a, b, c) : a + b + c = 0\}$

(a) $W = \{(a, b, c) : a = b = c\}$
Question 4. Solve the initial value problem $x' = Ax$ with

$$A = \begin{bmatrix} -2 & -2.5 \\ 10 & -2 \end{bmatrix}, \quad x(0) = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$
Question 5. Let $V = P(t)$, the vector space of real polynomials. Determine whether or not $W$ is a subspace of $V$. Explain.

(a) $W$ consists of all polynomials with integer coefficients.

(b) $W$ consists of all polynomials with degree $\geq 6$ and the zero polynomial.

(c) $W$ consists of all polynomials with only even powers of $t$. 
Question 6. (a) Find the matrix of fundamental solutions for the homogeneous system

\[
\begin{bmatrix}
  x' \\
  y'
\end{bmatrix}
= \begin{bmatrix} 1 & 1 \\
  0 & 1 \end{bmatrix}
\begin{bmatrix} x \\
  y
\end{bmatrix}
\]

(b) Find the general solution for the nonhomogeneous system

\[
\begin{bmatrix}
  x' \\
  y'
\end{bmatrix}
= \begin{bmatrix} 1 & 1 \\
  0 & 1 \end{bmatrix}
\begin{bmatrix} x \\
  y
\end{bmatrix}
+ \begin{bmatrix} e^t \\
  e^{-t}
\end{bmatrix}
\]
Question 7. (a) Find an invertible matrix $P$ and a diagonal matrix $D$ such that

$$A = PDP^{-1}, \quad A = \begin{bmatrix} 6 & -2 \\ 6 & -1 \end{bmatrix}$$

(b) Find $A^{-1}$ without directly inverting $A$. 
Question 8. Let $S$ consist of the following vectors in $\mathbb{R}^4$:

$$u_1 = (1, 1, 0, -1), \quad u_2 = (1, 2, 1, 3), \quad u_3 = (1, 1, -9, 2), \quad u_4 = (16, -13, 1, 3)$$

(a) Show that $S$ is orthogonal and a basis of $\mathbb{R}^4$.

(b) Find the coordinates of an arbitrary vector $v = (a, b, c, d)$ in $\mathbb{R}^4$ relative to the basis $S$. 